

PERCEPTION IS ANALOG: THE ARGUMENT FROM
WEBER'S LAW*

Many familiar representations can be classified as analog or digital. My watch has rotating hands that display an analog representation of the time, though my iPhone represents the time of day digitally. Mercury thermometers display analog representations of temperature, though most thermometers nowadays use digital representations. A guiding assumption of many philosophers and cognitive scientists is that the mind is representational. If that guiding assumption is correct, we should likewise be able to ask whether the mind's representations are analog or digital. While we can ask this question of any of the mind's representations, my interest here is in sensory perception. A number of philosophers have argued that perceptual representations are analog. Most prominent, perhaps, are a series of arguments from the 1980s, owing to Fred Dretske, Gareth Evans, and Christopher Peacocke.¹ While suggestive, these arguments have been subjected to forceful criticisms in the intervening years, leaving perception's status as analog uncertain. I want to revisit this issue from a fresh perspective that promises to make progress where earlier efforts have stalled.

Following a brief critical review of the earlier arguments, I will begin to develop a new argument that perception is analog which draws on Weber's Law, a well-entrenched finding in psychophysics. This new argument is an adaptation of a similar argument that cognitive scientists have leveraged in support of the contention that primitive numerical representations are analog. But I will work to deepen and

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¹ Fred I. Dretske, *Knowledge and the Flow of Information* (Cambridge, MA: MIT Press, 1981); Gareth Evans, *The Varieties of Reference*, ed. John McDowell (New York: Oxford University Press, 1982); Christopher Peacocke, "The Inaugural Address: Analogue Content," *Proceedings of the Aristotelian Society, Supplementary Volumes*, LX, 1 (July 1986): 1–17; Christopher Peacocke, "Perceptual Content," in J. Almog, J. Perry, and H. Wettstein, eds., *Themes from Kaplan* (New York: Oxford University Press, 1989), pp. 297–329; and Christopher Peacocke, *A Study of Concepts* (Cambridge, MA: MIT Press, 1992).

expand the argument in several ways. First, I will extend the argument beyond representations of number to representations of many other magnitudes, including luminance, sound-wave amplitude, pressure, weight, temperature, and various chemical concentrations. Second, whereas earlier discussions by cognitive scientists often locate analog representations in cognition, I will argue that perception proper is also analog. Third, prior discussants typically mischaracterize the sense of ‘analog’ in which it is true that Weber’s Law is indicative of analog representations. I will provide a more accurate characterization. Fourth, I will reply to two powerful objections that earlier discussions have largely ignored. Finally, I will explore whether the analog vehicles that Weber’s Law gives us reason to posit are located in conscious experience itself. To the best of my knowledge, no other discussions of the argument from Weber’s Law have broached this question.

I. PAST ARGUMENTS

I begin by evaluating the three arguments from the 1980s that purport to show that perception is analog. Because this is familiar ground, I will cover it relatively quickly.

I.1. Dretske. According to Dretske, all signals are “pregnant with information”; they carry many messages when they carry any.² For example, the sentence ‘The cup is red’ carries not only the message that the cup is red, but also any messages that are entailed by that message, such as the message that the cup is either red or blue. But if all signals are pregnant with information, analog signals are nine months long. For example, a picture of a red cup not only carries information about its color, but also about its size, shape, and orientation. Dretske argues that perception is likewise bursting with information. You cannot see a red cup without seeing it as having some specific shape, size, and orientation. You cannot *just* see a red cup. By contrast, you can form the belief that a cup is red without forming any beliefs about its size, shape, or other properties. Dretske infers that perception has an unprocessed analog format and that cognitive states such as beliefs are the product of an analog-to-digital conversion process that takes perceptual states as inputs.

Mohan Matthen objects that Dretske’s characterization of perception is at odds with what we know about perceptual processing.³ Drawing on empirical work, Matthen argues that, starting in the retina, stimuli are classified along separate dimensions—including spatial

²Dretske, *Knowledge and the Flow of Information*, *op. cit.*, p. 73.

³Mohan Matthen, *Seeing, Doing, and Knowing: A Philosophical Theory of Sense Perception* (New York: Oxford University Press, 2005).

frequency, length, orientation, hue, saturation, and brightness—and that each dimension of classification is represented in a “feature map” that plots the dimension’s distribution in physical space.⁴ When a perceiver attends to a specific location in physical space, the corresponding parts of the various feature maps are bound together. The result is the *illusion* of an unprocessed, analog experience.

What has happened is that the message has been assembled from digitized components in a prescribed form. The attended image seems, from the point of view of the perceiver, to be pictorial in character, to resemble the image projected on the retina. . . . Visual attention creates this illusion by reassembling the messages it has separately devised.⁵

The problem with Dretske’s argument, in other words, is not the observation that perception is nine-months pregnant, but the failure to appreciate that its pregnancy could be fathered by the assembly of various digital components.

I.2. Evans (and Heck). Evans famously asks, “Do we really understand the proposal that we have as many color concepts as there are shades of color that we can sensibly discriminate?”⁶ Richard Kimberly Heck develops Evans’s rhetorical question into *the richness argument*.⁷ Perception is *rich* because its content vastly outstrips the concepts we have to characterize it. Thus, while we have concepts to characterize some shades of red (*scarlet, crimson, mahogany*), we do not have concepts to characterize all the shades we perceive. Although Evans and Heck take this observation to support the conclusion that perception has non-conceptual content, it is not hard to see how their argument might be extended to support the conclusion that perception has an analog format. According to one way of understanding the concept *analog*, analog representations are (nearly) continuous or dense, enabling indefinitely many values to be represented along a given dimension.⁸ So if perception were analog its content might be expected to outstrip the content of any discrete, conceptual system of representation.

⁴Horace B. Barlow, “Single Units and Sensation: A Neuron Doctrine for Perceptual Psychology?,” *Perception*, 1, 4 (March 1972): 371–94; Anne Treisman, “The Binding Problem,” *Current Opinion in Neurobiology*, vi, 2 (April 1996): 171–78; and Richard H. Masland, “Vision: The Retina’s Fancy Tricks,” *Nature*, CDXXXIII, 6938 (May 2003): 387–88.

⁵Matthen, *Seeing, Doing and Knowing*, *op. cit.*, pp. 69–70.

⁶Evans, *The Varieties of Reference*, *op. cit.*, p. 229.

⁷Richard Kimberly Heck, “Nonconceptual Content and the ‘Space of Reasons’,” *The Philosophical Review*, CIX, 4 (October 2000): 483–523 (originally published under the name “Richard G. Heck, Jr”).

⁸Nelson Goodman, *Languages of Art: An Approach to a Theory of Symbols* (Indianapolis, IN: Hackett, 1968).

While there are many ways this argument might be (and has been) challenged, for our purposes I only want to draw attention to one. Several philosophers have targeted as a *non sequitur* the transition from the premise that the content of perception cannot be expressed using the concepts we have in thought to the conclusion that the content of perception cannot be expressed by any system of concepts.⁹ Because perception might have its own proprietary suite of concepts, the fact that a concept (for a determinate shade of red, say) is missing from thought does not mean that it cannot be present in perception. So the richness of perception does not show that perception has non-conceptual content or an analog format.

I.3. Peacocke. Peacocke observes that a person may perceive the distance between his bookshelf and his door without knowing what the distance is in feet and inches.¹⁰ And of course the limitation is not confined to imperial units; he may have no inkling of the distance in metric units, or any other familiar units. Nor is the phenomenon limited to distance. Other magnitude types (size, direction, speed, and so on) presented in perception also resist characterization in units. Peacocke thus infers that perception is *unit free*: it presents magnitudes without presenting them in any particular units.

Yet while analog representations are unit free, so are plenty of non-analog representations—proper names, for example. Thus, just as meteorologists name every hurricane, perception could in principle assign a name to every experienced magnitude. But Peacocke also observes that the perception of magnitudes is extremely fine grained. There is virtually no end to the manners by which various magnitudes can be experienced. Thus, to a first approximation, for any two distances (directions, sizes, and so on), d and d' , there is a manner of experience that includes d but not d' , and another manner of experience that includes d' but not d . It is thus implausible that there is a name for every manner of experiencing a magnitude type, and so it is reasonable to conclude that the perception of magnitudes has an analog format.

⁹Alex Byrne, "Perception and Conceptual Content," in E. Sosa and M. Steup, eds., *Contemporary Debates in Epistemology* (Oxford: Blackwell, 2005), pp. 231–50; Richard Kimberly Heck, "Are There Different Kinds of Content?," in B. McLaughlin and J. Cohen, eds., *Contemporary Debates in Philosophy of Mind* (Oxford: Blackwell, 2007), pp. 117–38 (originally published under the name "Richard G. Heck, Jr."); Matthen, *Seeing, Doing, and Knowing*, *op. cit.*, chapter 3; Jeff Speaks, "Is There a Problem about Non-conceptual Content?," *The Philosophical Review*, cxiv, 3 (July 2005): 359–98.

¹⁰Peacocke, "Analogue Content," *op. cit.*; Peacocke, "Perceptual Content," *op. cit.*; and Peacocke, *A Study of Concepts*, *op. cit.*

But while it is surely true that perception does not use feet and inches—or any other set of familiar units—it is less obvious that it lacks its own units.¹¹ Peacocke's claim to the contrary is grounded in introspection, but if introspection cannot settle whether perception has proprietary color concepts, it is not clear how it can settle whether perception has proprietary units. It is not at all straightforward how proprietary units might be expected to manifest in experience.

1.4. Clarifications. The objections I have rehearsed to these three arguments are not intended to be original. Nor are they intended to be decisive. There is undoubtedly much more that could be said in defense of each argument. Rather, my intention is simply to contextualize the new argument I will be presenting below and to help motivate its consideration.

Before I begin to develop the new argument, two further clarifications are in order. First, there is a question of what makes perception the right sort of thing to have a format, and thus be analog (or digital). On the standard view, which I will assume going forward, it is representational *vehicles*—causally efficacious bearers of representational content—that have a format. To say that perception is analog is thus to say that it has analog vehicles. An alternative view, endorsed by Evans and Peacocke, understands format at the level of *content*; to say that perception is analog is to say it has analog content. I have argued elsewhere that the difference between these two views is less significant than it might seem since the format of a mental state's content tends to mirror the format of its vehicle.¹² Thus, perceptual content is likely to be analog if, and only if, perceptual vehicles are analog. But I will not rely on that point here. For the purposes of developing my argument below, I will understand the thesis that perception is analog solely as a claim about perception's vehicles.

Second, like Peacocke, I will argue that perception is analog because of the way in which it represents *magnitudes* such as distance. This argument is compatible with the possibility that perception represents non-magnitudes—objects, say—in a way that is not analog. It is also compatible with Matthen's important observation that experience is not raw and unprocessed, but rather consists in the

¹¹ One possibility (though by no means the only possibility) is that perception uses bodily units to represent magnitudes. For example, David J. Bennett, "How the World Is Measured Up in Size Experience," *Philosophy and Phenomenological Research*, LXXXIII, 2 (September 2011): 345–65, suggests that vision uses eye-level units to represent size, which would explain why one's childhood bedroom looks smaller than remembered when one views it as an adult.

¹² Jacob Beck, "The Generality Constraint and the Structure of Thought," *Mind*, cxxxi, 483 (July 2012): 563–600. See also Heck, "Are There Different Kinds of Content?," *op. cit.*

assembly of various components. My contention is only that the product of all of that processing is analog with respect to its representation of magnitudes.

II. EMPIRICAL BACKGROUND

I will assume that perception represents a wide range of *objective magnitudes*—magnitudes that have a home in a non-mental science such as physics or chemistry. For example, vision represents luminance (light intensity); audition represents sound-wave amplitude; touch represents pressure, weight, and temperature; smell and taste represent concentrations of chemicals such as acetic acid, sucrose, and salt. There are also objective magnitudes that are represented by perception in multiple modalities, including length, area, volume, distance, number, and duration.

I am not putting a lot of weight on the term ‘represent’ here. While I do mean that (within familiar and soon-to-be-discussed limits) perception systematically tracks, and enables the discrimination of, objective magnitudes, I do not mean that objective magnitudes capture the “mode of presentation” of perception. For example, I do not mean that audition presents sounds *as* having this or that amplitude, or that taste presents foods *as* having this or that sucrose concentration. Nor do I mean that perception represents objective magnitudes only in virtue of the application of constancy mechanisms (though often constancy mechanisms are involved).¹³ I also do not mean to deny that perception represents non-objective magnitudes in addition to objective magnitudes. Thus, although I will continue to use the term ‘represent’ to describe the relation that perception stands in to objective magnitudes, one could substitute another term, such as ‘register’ or ‘track’, without loss. Similarly, one could replace my use of ‘perception’ with ‘perception or sensory registration’, though I will stick to ‘perception’ for economy of expression.

Evidence that observers represent objective magnitudes comes from experiments that hold all else constant while varying these magnitudes and asking questions such as, “Is stimulus A the same as stimulus B?” or “Which stimulus is more intense, A or B?” The results not only demonstrate that perceivers have the ability to discriminate the objective magnitudes listed above, but also that their ability to discriminate those magnitudes obeys what is known as *Weber’s Law*: $\Delta M/M = k$, where M is the value of a magnitude (such as luminance), ΔM is the

¹³For an argument that constancies are essential to perceptual representation, see Tyler Burge, *Origins of Objectivity* (New York: Oxford University Press, 2010).

minimal change in the value of the magnitude required to bring about a just-noticeable difference on the part of the subject, and k is a constant known as a Weber fraction. For example, suppose you are presented with the luminance of 20 candles, and it takes the addition of two further candles for you to notice a difference in luminance. Your Weber fraction for luminance would then be $1/10$, which can be used to predict just-noticeable-difference values for any luminance value. For example, if you were presented with 10 candles, we would need to add one more candle for you to notice a luminance difference; if you were presented with 50 candles, we would need to add five candles for you to notice the difference; and so on. Different magnitudes are associated with different Weber fractions, but every magnitude listed in the first paragraph of this section (and many more besides) is associated with some constant Weber fraction or other across the majority of the range of detectable magnitudes.¹⁴

Weber's Law depends on the notion of a *just-noticeable difference*, which psychophysicists operationalize by selecting as a criterion some fixed percentage of trials—say, 75%—on which subjects must correctly judge whether two stimuli differ. Altering this criterion alters k . For example, reducing the criterion from 75% to 65% lowers k , and increasing the criterion from 75% to 85% increases k . Because the selection of any particular criterion is somewhat arbitrary, the precise value of k for any given magnitude is somewhat arbitrary. But the sense in which k is not arbitrary is this: once a criterion is selected, k remains constant over a wide range of values and is thus predictive of ΔM for any M within that range.¹⁵

¹⁴A brief explanation of Weber's Law can be found in any psychophysics textbook (usually the first chapter)—for example, Jeremy M. Wolfe et al., *Sensation and Perception*, 4th ed. (New York: Sinauer Associates, 2015), p. 6; or E. Bruce Goldstein, *Sensation and Perception*, 9th ed. (Toronto: Wadsworth, 2013), p. 14. Weber's Law tends to break down at the extremes—that is, near the upper and lower limits of detectability. There is also some controversy about whether it is more accurately characterized as $\Delta M / (M + c) = k$, where c is a small constant, perhaps corresponding to background sensory noise that is always present (George A. Gescheider, *Psychophysics: The Fundamentals*, 3rd ed. (Mahwah, NJ: Lawrence Erlbaum, 1997), pp. 6–7; Donald R. J. Laming, *Sensory Analysis* (London: Academic Press, 1986), pp. 6–8; and Donald Laming, *The Measurement of Sensation* (New York: Oxford University Press, 1997), pp. 36–37). And there are questions about whether the discrimination of certain magnitudes (such as pure-tone amplitudes) closely approximates Weber's Law without following it to a T (Gescheider, "Psychophysics," *op. cit.*, pp. 8–10; Laming "Sensory Analysis," *op. cit.*, pp. 13–17). These exceptions, if that is what they are, remind us that we are dealing with a special science, but should not impact the arguments to come.

¹⁵Another sense in which the precise value of k is somewhat arbitrary is that it can vary with the experimental setup—the size of the stimuli, the duration of their presentation, the properties of the background, and so on. But again, once the experimental setup is

Weber's Law indicates that the ability to discriminate two magnitudes is determined by their ratio. As the ratio approaches 1:1, the ability to discriminate the magnitudes decreases. Here is another way to appreciate this point. When an observer is presented with a sample magnitude value and then asked whether various test values are the same as the sample or not, her percentage of "same" judgments will form a normal (Gaussian) distribution that peaks when the test value is identical to the sample value. Thus, associated with each value for any given magnitude is a bell curve that captures how well an observer can discriminate that value from its neighbors. Interestingly, these bell curves change their shapes in a very predictable way. The curves flatten and widen in proportion to the value of the objective magnitude (Figure 1). In statistical terms, the standard deviation (σ), a measure of the spread of the distribution, increases linearly with the magnitude value—a property known as *scalar variability*.¹⁶ As Figure 1 illustrates, the curves overlap more as the magnitude value increases. In fact, the area of overlap increases in direct proportion to the ratio of the

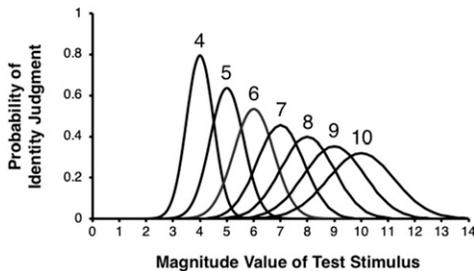


Figure 1. The x -axis represents the magnitude value of the test stimulus in arbitrary units on a linear scale, and the y -axis represents the probability that the subject judges the test stimulus to be identical to the sample stimulus. Curves are shown for sample stimuli of 4 to 10 units only. Figure courtesy of Justin Halberda.

fixed h remains constant over a wide range of values and is thus predictive of ΔM for any M within that range.

¹⁶John Gibbon, "Scalar Expectancy Theory and Weber's Law in Animal Timing," *Psychological Review*, LXXXIV, 3 (May 1977): 279–335; and C. R. Gallistel and Rochel Gelman, "Preverbal and Verbal Counting and Computation," *Cognition*, XLIV, 1–2 (January 1992): 43–74.

magnitude values they correspond to, explaining why discrimination deteriorates in direct proportion to magnitude ratio. For example, the area of overlap between the 4 and 5 curves is identical to the area of overlap between the 8 and 10 curves. Unlike Weber's Law, this explanation of ratio sensitivity does not invoke the notion of a just-noticeable difference, and so requires no appeal to an arbitrary criterion. But given σ and the (empirically supported) assumption of a Gaussian distribution, one can predict performance at any given criterion. Scalar variability is thus arguably more fundamental than Weber's Law. Nevertheless, the two phenomena are clearly closely related, and it is common to use the term 'Weber's Law' to refer to them both, as I will do here.¹⁷

III. THE ARGUMENT FROM WEBER'S LAW

Weber's Law is indicative of analog representations. To see why, an analogy is helpful. Suppose you are watching a basketball game between the Cavaliers and the Warriors. To keep score, you adopt an unorthodox method that relies on two cylindrical buckets, one to track the Cavaliers' score, and another to track the Warriors' score. You also have a cup, and every time the Cavaliers or Warriors score a point, you pour one cup of water into the corresponding bucket. Because the buckets are cylindrical, the height of the water in each bucket is a linear function of the corresponding team's score, and you use the height of the water in each bucket as a representation of each team's score. Intuitively, these representations are analog since the height of the water in each bucket is a direct analog of the total points that each team scores. As the score goes up, so does the height of the water. At the end of the game, you determine the winner by comparing the water heights in each bucket.

These bucket representations might be subject to systematic noise. The water might slosh around, interfering with comparisons. Or unbeknownst to you, one bucket might leak more than the other, or have a greater circumference than the other. Even with such noise, it will be easy enough to tell who won if the game is a blowout. But if it is your typical NBA game, with each team scoring over 100 points and the

¹⁷Justin Halberda and Darko Odic, "The Precision and Internal Confidence of Our Approximate Number Thoughts," in D. Geary, D. Berch, and K. Mann Koepke, eds., *Evolutionary Origins and Early Development of Number Processing* (New York: Academic Press, 2014), pp. 305–33, helpfully discuss these issues, though they recommend conceiving of the Weber fraction as referring to yet a third phenomenon: "a scaling factor for determining internal variability" (*ibid.*, p. 320). I think this invites confusion of the explanandum (scalar variability in discriminative *behavior*) with the explanans (scalar variability in some *internal mental or neural response*), and so I reserve the terms 'Weber's Law' and 'Weber fraction' to describe behavioral phenomena.

game being decided by a buzzer-beater, the method will be much less reliable.

Weber's Law dictates that comparisons are ratio sensitive, and the bucket system illustrates why ratio sensitivity might emerge from comparisons of analog representations that are accompanied by noise. To get Weber's Law exactly, however, we cannot pair just any score-to-height mapping with any pattern of noise. The mapping and the noise need to complement each other in the right way. For example, if the mapping is linear (as when the buckets are cylindrical), then the noise needs to increase in direct proportion to the score (say, because one bucket has a larger circumference than the other). Alternatively, if the mapping is logarithmic (imagine a bucket whose radius, rather improbably, increases almost exponentially with the height), then constant noise (say, because one bucket had a little residual water in it when the game began) would generate Weber's Law.

In either case, Weber's Law is explained only because similarities among the magnitudes represented (such as the scores) are mirrored by similarities among the representations (such as the water heights). Just as a score of 40 points is more similar to a score of 50 points than to a score of 60 points, a height of 40 cm is more similar to a height of 50 cm than to a height of 60 cm. Thus, it is hard to determine the winning team when the scores are similar because the water heights are also similar and there is noise in the system. But note that digital representations do not have this property. For example, the Arabic numeral '40' is *not* more similar to the Arabic numeral '50' than to the Arabic numeral '60'.

We can illustrate the point that digital representations should not be expected to give rise to Weber's Law by considering how noise would affect binary representations of objective magnitudes. If the noise were such that each digit in a binary representation had an equal probability of being corrupted (such that a 0 becomes a 1 or a 1 becomes a 0), random large errors would be about as likely as random small errors, contradicting the ratio sensitivity of Weber's Law. For example, the representation '10001' of the relatively large intensity 17 would be just as likely to morph into the representation '10000' of the adjacent intensity 16 as into the representation '00001' of the tiny intensity 1. Additionally, error patterns would exhibit systematic biases that were independent of ratio, which is also at odds with Weber's Law. For example, '10001' would be twice as likely to morph into a representation of its predecessor, '10000', as into a representation of its successor, '10010', since the latter would require two digits to be corrupted rather than one. Is there no *ad hoc* way to tamper with the

noise to make binary representations give rise to Weber's Law? Presumably there is. But the point is that it would take some *ad hoc* tampering. With analog representations, very natural patterns of noise generate Weber's Law.

Let us call the argument I have been developing that perceptual representations are analog *the argument from Weber's Law*. This argument has the form of an inference to the best explanation. If we assume that the mind represents objective magnitudes by way of mental magnitudes that are a direct analog of the objective magnitudes, and also posit the appropriate pattern of noise (any natural system of representation will inevitably contain some noise), we can provide a simple and elegant explanation of Weber's Law. For example, if we assume that the mental magnitude is a linear function of the objective magnitude and that the noise increases in proportion to the magnitude's value, Weber's Law falls out as a consequence. Alternatively, Weber's Law can be derived from the assumption that the mental magnitude is a logarithmic function of the objective magnitude and that the noise is constant. By contrast, if we do not assume that perceptual representations are analog, no comparably simple or elegant explanation is forthcoming. For example, if we assume that the mind represents objective magnitudes by way of arbitrary digits, there is no correspondingly simple way to introduce noise into the system such that Weber's Law emerges as a consequence. We thus have (defeasible, a posteriori) justification for believing that perceptual representations are analog. There is some mental magnitude, comparable to the height of water in the bucket system, that is a direct analog of the objective magnitudes discriminated by the perceptual system.

As I noted in the introduction, the argument from Weber's Law is not unprecedented. A number of psychologists have advanced similar arguments.¹⁸ Some philosophers have discussed the argument as well,

¹⁸Susan Carey, *The Origin of Concepts* (New York: Oxford University Press, 2009), pp. 118–35; Stanislas Dehaene, *The Number Sense: How the Mind Creates Mathematics*, rev. ed. (New York: Oxford University Press, 2011), pp. 17–20; Gallistel and Gelman, "Preverbal and Verbal Counting and Computation," *op. cit.*; C. R. Gallistel and Rochel Gelman, "Non-verbal Numerical Cognition: From Reals to Integers," *Trends in Cognitive Sciences*, iv, 2 (February 2000): 59–65; and C. R. Gallistel, Rochel Gelman, and Sara Cordes, "The Cultural and Evolutionary History of the Real Numbers," in S. C. Levinson and P. Jaisson, eds., *Evolution and Culture: A Fyssen Foundation Symposium* (Cambridge, MA: MIT Press, 2006), pp. 247–74.

at least in passing.¹⁹ But these previous discussions of the argument focus almost exclusively on primitive representations of number—for example, the rat’s ability to represent the number of times it has pressed a lever, or the human infant’s ability to represent the number of dots on a screen. Very occasionally, representations of temporal duration or rate are discussed as well. This narrow focus obscures the fact that Weber’s Law applies much more widely. In fact, Weber’s Law has its original home in perception, where (as noted above) it characterizes the discrimination of such variegated magnitudes as luminance, sound-wave amplitude, pressure, weight, temperature, and myriad chemical concentrations. It is thus worth exploring whether the argument might be expanded to encompass all of these perceptual magnitudes.

That the argument from Weber’s Law might show that *perception* is analog has been further obscured by a tendency to classify primitive numerical representations as post-perceptual. For example, Susan Carey locates analog numerical representations in what she calls “core cognition” rather than in perception proper.²⁰ While I agree with Carey that analog representations of number occur in cognition,²¹ for reasons that will become clear below I think that they also occur in perception.²²

The argument from Weber’s Law invites various objections, particularly when it is extended to perception in the way that I am proposing. I will consider what I take to be the two objections with the most *prima facie* force. The first, more radical objection maintains that the world itself can explain Weber’s Law, and so we do not need to

¹⁹ Brian Ball, “On Representational Content and Format in Core Numerical Cognition,” *Philosophical Psychology*, xxx, 1–2 (January 2017): 119–39; Beck, “The Generality Constraint and the Structure of Thought,” *op. cit.*; Jacob Beck, “Analogue Magnitude Representations: A Philosophical Introduction,” *The British Journal for the Philosophy of Science*, LXVI, 4 (December 2015): 829–55; Jacob Beck, “Analog Mental Representation,” *WIREs Cognitive Science*, ix, 6, (December 2018): e1479; Burge, *Origins of Objectivity*, *op. cit.*, pp. 472–83; Matthew Katz, “Mental Magnitudes and Increments of Mental Magnitudes,” *Review of Philosophical Psychology*, iv, 4 (December 2013): 675–703; Matthew Katz, “Analog Representations and Their Users,” *Synthese*, cxxiii, 3 (March 2016): 851–71; and Stephen Laurence and Eric Margolis, “Number and Natural Language,” in P. Carruthers, S. Laurence, and S. Stich, eds., *The Innate Mind: Structure and Contents* (New York: Oxford University Press, 2005), pp. 216–35.

²⁰ Carey, “The Origin of Concepts,” *op. cit.*

²¹ Beck, “The Generality Constraint and the Structure of Thought,” *op. cit.*, pp. 585–89.

²² That the same properties can be represented in perception and cognition should not be controversial. You can represent redness in visual perception and also in imagination, memory, and thought. For the specific point that perception and *core* cognition can represent some of the same properties, see Tyler Burge, “Border Crossings: Perceptual and Post-perceptual Object Representation,” *Behavioral and Brain Sciences*, xxxiv, 3 (June 2011): 125.

appeal to anything analog in the mind. Two objective magnitudes are inherently more similar, and thus harder to discriminate, as their ratio approaches 1:1. Thus, one might expect any system of representation, whether analog or not, to have more difficulty discriminating two magnitudes as their ratio approaches 1:1. Call this the *world objection*.

The second objection locates the source of Weber's Law neither in the world nor in the format of perceptual representation, but in the process of converting sensory information from the world into (possibly digital) representations. If that process exhibits the right kind of noise, discrimination should be ratio sensitive independently of the format of representation. Call this the *conversion objection*.

I will address the world and conversion objections in sections v and vi, respectively. First, however, it will help to have a clearer understanding of the concept *analog* that the argument from Weber's Law presupposes.

IV. ANALOG REPRESENTATION

What makes a representation *analog*? The scholarly literature contains a staggering diversity of accounts, but with only a little spit and pressure most of them can be squeezed into two broad bins: one that revolves around the idea that analog representations are *continuous* (or approximately so) rather than discrete or differentiated, and a second that centers on the idea that analog representations *mirror* (for example, are isomorphic to, or bear some other structure-preserving mapping toward) what they represent.²³ While each conception fruitfully characterizes certain representations, and many paradigmatic

²³For some prominent accounts that belong in the continuous bin, see Goodman, *Languages of Art*, *op. cit.*; John Haugeland, "Analog and Analog," *Philosophical Topics*, xxii, 1 (Spring 1981): 213–25; and Whit Schonbein, "Varieties of Analog and Digital Representation," *Minds and Machines*, xxiv, 4 (November 2014): 415–38. For accounts that are better fits for the mirroring bin, see James Blachowicz, "Analog Representation beyond Mental Imagery," this JOURNAL, xciv, 2 (February 1997): 55–84; John Kulvicki, "Analog Representation and the Parts Principle," *Review of Philosophy and Psychology*, vi, 1 (March 2015): 165–80; David Lewis, "Analog and Digital," *Noûs*, v, 3 (September 1971): 321–27; Corey Maley, "Analog and Digital, Continuous and Discrete," *Philosophical Studies*, clv, 1 (August 2011): 117–31; Christopher Peacocke, "Magnitudes," in *The Primacy of Metaphysics* (Oxford: Oxford University Press, 2019); and Roger N. Shepard and Susan Chipman, "Second-Order Isomorphism of Internal Representations: Shapes of States," *Cognitive Psychology*, 1, 1 (January 1970): 1–17. Dretske's account of analog representation is anomalous; it resists categorization in either bin. According to his official definition (Dretske, *Knowledge and the Flow of Information*, *op. cit.*, p. 137), a signal with the content that *a* is F is analog just in case it carries additional information about *a* beyond that "nested" in *a*'s being F. This definition is relativized to a message, and has the counterintuitive consequence that all signals carry analog messages and digital messages. For example, the sentence 'The letter is an A' not only carries the digital message that the letter is an A, but also the analog message that the letter is either an A or a B. Likewise, a picture of the letter A not only carries the analog message that the letter is an A, but also

analog representations answer to both, they are not coextensive.²⁴ Watches and clocks often have rotating hands that advance in discrete steps and are thus not analog in the continuous sense, but they are still analog in the mirroring sense since the hand's angle mirrors the time of day.

Several authors mistakenly take the argument from Weber's Law to establish that mental representations are analog in the continuous sense. For example, Stanislas Dehaene reports that the argument from Weber's Law shows that the nervous system "does not seem to be able to count using discrete tokens," implying that it must use continuous vehicles instead.²⁵ C. R. Gallistel and Rochel Gelman likewise take primitive numerical representations to be analog in the continuous sense, which leads them to conclude that they stand for continuous contents—the real numbers.²⁶ Stephen Laurence and Eric Margolis²⁷ and Tyler Burge²⁸ criticize Gallistel and Gelman for assuming that continuous vehicles need to have continuous contents, but they appear to join Gallistel and Gelman in assuming that Weber's Law is indicative of continuous vehicles. For instance, Burge writes that the representation of primitive numerical magnitudes "seems to be analog or continuous rather than discrete."²⁹ It is, however, simply a mistake to suppose that Weber's Law is evidence of continuous vehicles—a mistake that likely arises from conflating the continuous and mirroring conceptions of analog representation. Weber's Law is evidence of analog representations in the mirroring sense only. To see this, note that in the bucket system it is because similarities among water heights mirror similarities among basketball scores that the potential to conform to Weber's Law emerges. Although water is a continuous medium and height can be measured continuously, those facts are not essential to understanding why the system obeys Weber's Law. We could replace the water with pebbles or measure height in millimeter increments without altering the basic explanation. Given the right kind of noise, all that matters is that there is some magnitude (such as water height, water volume,

the digital message that consists of the conjunction of all of its analog messages. It is hard to see how this definition could even begin to explain Weber's Law.

²⁴ Lewis, "Analog and Digital," *op. cit.*

²⁵ Dehaene, *The Number Sense*, *op. cit.*, p. 19.

²⁶ Gallistel and Gelman, "Non-verbal Numerical Cognition: From Reals to Integers," *op. cit.*; and Gallistel, Gelman, and Cordes, "The Cultural and Evolutionary History of the Real Numbers," *op. cit.*

²⁷ Laurence and Margolis, "Number and Natural Language," *op. cit.*

²⁸ Burge, *Origins of Objectivity*, *op. cit.*, pp. 480–81.

²⁹ *Ibid.*, p. 477.

pebble weight, or pebble number) that mirrors the magnitude that is represented (the score). Similarly, in order to explain why perceptual discriminations obey Weber's Law, we need to assume that objective magnitudes are represented by mental magnitudes such that similarities in the former are mirrored by similarities in the latter. It does not matter if the mental magnitudes are composed from discrete elements or measured in increments. For example, we could explain Weber's Law if objective magnitudes were represented by neural firing rates (a continuous magnitude). But we could also explain Weber's Law if objective magnitudes were represented by the total number of neurons firing above some threshold within a given population (a discrete magnitude).

The argument from Weber's Law is thus committed to the mirroring conception of analog representation. But how should the mirroring conception be understood? According to Carey, analog representations are *iconic*; their parts represent part of what the representation as a whole represents.³⁰ A picture of a horse is iconic because parts of the picture represent parts of the horse (one part represents its tail, another part represents its head, and so on). The parts of the word 'horse', by contrast, do not represent parts of a horse, and so words are not iconic. Carey claims that primitive numerical representations are iconic, and thus analog.

Analog representations of number represent as would a number line—the representation of two (____) is a quantity that is smaller than and is contained in the representation for three (____).³¹

But this suggestion is confused. When a picture represents a horse, one thing with spatial properties (a picture) represents another thing with spatial properties (a horse), and so talk of "parts" makes sense. But as Brian Ball points out in a discussion of Carey's proposal, numbers themselves do not have parts.³² For example, the number two is certainly not a spatial part of the number three, and it is not obviously a "part" in any non-spatial sense either. Sam Clarke makes a similar point with respect to velocity: one can perceive the speed of a moving object, but speeds do not have parts (20 mph is not a part of

³⁰ Carey, *The Origin of Concepts*, *op. cit.* On the concept of iconic representation, see also Jerry Fodor, "The Revenge of the Given," in McLaughlin and Cohen, eds., *Contemporary Debates in Philosophy of Mind*, *op. cit.*, pp. 105–16; Stephen M. Kosslyn, *Image and Mind* (Cambridge, MA: Harvard University Press, 1980), pp. 33–35; and Kulvicki, "Analog Representation and the Parts Principle," *op. cit.*

³¹ Carey, *The Origin of Concepts*, *op. cit.*, p. 458.

³² Ball, "On Representational Content and Format in Core Numerical Cognition," *op. cit.*, pp. 131–32.

25 mph).³³ The point transfers to many other perceptible magnitudes, such as chemical concentrations. Furthermore, it is doubtful whether the neural code responsible for representing magnitudes has parts in the relevant sense. Suppose, for example, that numbers are represented by neural firing rates. As Clarke³⁴ and Peacocke³⁵ observe, a firing rate of 30 Hz does not have a firing rate of 20 Hz as a “part” in any standard interpretation of that term. And even if there were some extended (perhaps metaphorical) interpretation of ‘part’ such that 20 Hz counted as a “part” of 30 Hz, there is no reason to suppose that greater numbers would need to be represented by *greater* firing rates. Greater numbers could just as well be represented by *lesser* firing rates.³⁶ To accommodate this possibility, Carey would need a sense of ‘part’ that is symmetric, such that 20 Hz is a “part” of 30 Hz and 30 Hz is a “part” of 20 Hz. But any such sense would stretch our concept of a part beyond recognition. It is thus doubtful that Carey’s appeal to iconicity can helpfully elucidate the sense of ‘analog’ presupposed by the argument from Weber’s Law.

A more fruitful way to think about the mirroring conception of ‘analog’ is in terms of covariation.³⁷ Analog representation, in the relevant sense, involves the representation of one magnitude by a second magnitude such that the second magnitude has the function of increasing or decreasing with the first. Just *how* must the two magnitudes covary? It would be too strict to require a linear mapping. As we have seen, Weber’s Law could be explained by a logarithmic mapping together with constant noise, and we should not exclude analog representations from encoding logarithmic mappings. In fact, we should even allow that analog representations in the mirroring sense (as opposed to the continuous sense) can encode step functions. Watches with rotating hands are intuitively classified as analog even when the hand’s angle increases as a step function of time. Such step functions are at least monotonic, and so we might propose that analog representation in the mirroring sense requires a monotonic relationship between the represented and representing magnitudes. But for present purposes a firm commitment is not required. Even without a precise analysis of the kinds of covariation that are compatible with the

³³ Sam Clarke, “Beyond the Icon: Core Cognition and the Format of Perception,” unpublished manuscript.

³⁴ *Ibid.*

³⁵ Peacocke, “Magnitudes,” *op. cit.*

³⁶ Beck, “Analogue Magnitude Representations,” *op. cit.*, p. 838.

³⁷ For a congenial analysis, see Maley, “Analog and Digital, Continuous and Discrete,” *op. cit.* See also Beck, “Analogue Magnitude Representations,” *op. cit.*; Beck, “Analog Mental Representation,” *op. cit.*; and Peacocke, “Magnitudes,” *op. cit.*

mirroring conception, I think that we have a clear enough grasp of the general idea to get along with for now.

Earlier proponents of the thesis that perception is analog were not always operating with a covariational (or even mirroring) conception of analog representation. Does that mean that I am guilty of changing the subject? Not if perception is a natural kind. Whether and how it is analog should then be a matter for empirical discovery. It should not be determined by the imposition of a favored concept from some other context. That Weber's Law is best explained by representations that are analog in *any* recognizable sense is notable.

V. THE WORLD IS NOT ENOUGH

Any two objective magnitudes of a given type, such as luminance or number, will in one sense grow more similar as their ratio approaches 1:1. Thus, one might expect any system of representation, whether analog or digital, to have more difficulty discriminating two objective magnitudes as their ratio approaches 1:1. And so it might seem that Weber's Law can be explained by facts about the world itself, obviating the need to appeal to analog representations or anything else in the mind. In the case of luminance, for example: perhaps observers obey Weber's Law because the luminance from ten candles is more physically similar to the luminance from nine candles than to the luminance from eight candles, and so any process that converts luminance information in the world into a representation in the mind (even a digital one) is more likely to confuse the luminance from ten and nine candles than from ten and eight candles. In the case of number: perhaps observers obey Weber's Law because ten items are more physically similar to nine items than to eight items, and so any process that converts numerical information in the world into a representation in the mind (even a digital one) is more likely to confuse ten and nine items than ten and eight items.

This objection rightly points out that any perceptual system, whether analog or digital, should be more likely to confuse physically similar magnitudes than physically dissimilar magnitudes. But the objection fails to explain the unique shape of Weber's Law—for example, why variability is scalar rather than a square-root function. There are many ways nature could build systems that discriminate physical magnitudes, and there is simply no *a priori* reason to expect that Weber's Law, with its specific quantitative predictions, would be an inevitable outcome of any one of them. In particular, there is no *a priori* reason to expect ΔM and σ to increase as a constant, linear function of the magnitude value.

One way to appreciate this point is to notice that although there is a sense in which the similarity between two objective magnitudes increases as their ratio approaches 1:1, there is also a sense in which it does not. For example, although the ratio of 1:2 is further from 1:1 than the ratio of 55:100, the difference between 1 and 2 is just 1 whereas the difference between 55 and 100 is 45. Thus, if we think of similarity among objective magnitudes in terms of differences rather than ratios—that is, arithmetically rather than geometrically—objective magnitudes of 1 and 2 units are actually more similar than objective magnitudes of 55 and 100 units. And yet Weber's Law indicates that perceptual systems treat magnitudes of 55 and 100 units as more similar than magnitudes of 1 and 2 units. That surely tells us something interesting and contingent about the mind—namely, that it calculates similarities among magnitudes geometrically rather than arithmetically. And the assumption that magnitude representations are analog allows us to naturally and elegantly explain why that would be so.

For the specific magnitude of luminance, another way to appreciate the point that facts about the world cannot explain Weber's Law is to think about how the discriminations of a light detector limited only by photon noise—a so-called *ideal light detector*—would pattern. Because of the statistical laws governing photons, the standard deviation would increase with the square root of the luminance. As it happens, this is exactly what one finds in human observers, *but only near the absolute threshold*, the minimal luminance levels that humans can detect.³⁸ This finding, known as the DeVries-Rose (or Square Root) Law, suggests that at very low luminance levels the variability in discriminations of human observers *is* explained by the world. It is explained by physical facts about light (in particular, photon noise). But at modestly higher levels, human observers exhibit scalar variability instead; the standard deviation in their discriminative responses is a linear (not square-root) function of luminance. At these modestly higher luminance levels, the variability in human discriminations increases at a *greater* clip than one would predict from physical facts about light itself, and thus appears to be driven by the mind. And again, there is a very natural explanation why discriminations exhibit such scalar variability on the assumption that the representations underlying those discriminations are analog.

³⁸ H. B. Barlow, "Increment Thresholds at Low Intensities Considered as Signal/Noise Discriminations," *The Journal of Physiology*, cxxxvi, 3 (May 1957): 469–88; H. L. de Vries, "The Quantum Character of Light and Its Bearing upon Threshold of Vision, the Differential Sensitivity and Visual Acuity of the Eye," *Physica*, x, 7 (July 1943): 553–64; and Albert Rose, "The Sensitivity Performance of the Human Eye on an Absolute Scale," *Journal of the Optical Society of America*, xxxviii, 2 (1948): 196–208.

For some other magnitudes, the idea that Weber's Law is a product of the idiosyncratic structure of human and animal minds is further supported by evidence that certain effects, predicted by Weber's Law, occur in the absence of sensory stimulation, when subjects are engaged in offline mental operations. This evidence also speaks against the conversion objection, and so I will consider it in the next section along with my reply to that objection.

VI. WEBER'S LAW WITHOUT CONVERSION

Ordinary scorekeepers do not rely on buckets and measuring cups. They continually update digital scoreboards. The job is not difficult, and so errors are rare. But it is not hard to imagine how systematic errors might be introduced. The scorekeeper might be intoxicated, distracted, or disposed toward frequent bathroom breaks. The button she presses to update the scoreboard might be unreliable, sometimes adding several points when it is pressed, and sometimes adding none. In short, the process by which she updates the scoreboard could be subject to noise even if the scoreboard's representations are digital. As a result, the scoreboard might be an unreliable indicator of the winning team, especially as the ratio of the scores approaches 1:1. Given the right pattern of noise, discriminations of the winning team that were based on the scoreboard would even conform to Weber's Law. Hence the conversion objection: Weber's Law can be explained by a noisy conversion process that takes sensory information as inputs and yields digital perceptual representations as outputs.

This objection is not without force. But it also makes a specific prediction: eliminate the noisy conversion process and you should eliminate any signs of Weber's Law. This prediction is not borne out. To illustrate, I will discuss some findings from the perception of numerical magnitudes. Although I am ultimately eager to expand the argument from Weber's Law beyond numerical magnitudes, this is one place where work on numerical magnitudes proves illuminating. It also provides a welcome opportunity to present evidence that numerical magnitudes are represented in perception and not just cognition.

As I indicated in section iv, perceivers are able to discriminate numerical magnitudes in multiple modalities. One such modality is vision. If you are quickly presented with two displays of dots, you can, within limits imposed by Weber's Law, determine which display has more dots even if you do not have time to explicitly count them. And you can do this even when other dimensions are controlled for, such as

total dot area or dot texture density.³⁹ One reason to think that numerical information is being perceived (and not just cognized) in these studies is that they give rise to adaptation effects.⁴⁰ As David Burr and John Ross have shown, staring at a large number of dots for an extended duration leads perceivers to underestimate the number of dots on a subsequent display; and staring at a small number of dots for an extended duration leads perceivers to overestimate the number of dots on a subsequent display.⁴¹ This effect includes a strong conscious component. Following adaptation, subsequent dot displays really *look* to be more or less numerous.⁴²

On their own, these adaptation effects are not sufficient to establish that numerical information is perceived. At least two further things must be shown. The first is that the adaptation effects are due to number and not some covarying property. While this is a live empirical issue, researchers have gone to great lengths to control for covarying properties. For example, there is compelling evidence that adaptation is sustained when contrast, dot size, and texture density are controlled for, suggesting that it really is number that is being tracked.⁴³ This conclusion is also consistent with the phenomenology. As Burr and Ross report, “One of the more fascinating aspects of this study—as readers can verify for themselves with the online demonstration—is that although the total apparent number of dots is greatly reduced after adaptation, no particular dots seem to be missing.”⁴⁴ Numerical phenomenology thus appears to float free of the phenomenology of the dots’ lower-level properties, at least to some extent.

Second, it needs to be shown that the adaptation is genuinely perceptual and not what Ned Block calls “the ‘cognitive phenomenology’

³⁹ Hilary Barth, Nancy Kanwisher, and Elizabeth Spelke, “The Construction of Large Number Representations in Adults,” *Cognition*, LXXXVI, 3 (January 2003): 201–21; and S. L. Franconeri, D. K. Bemis, and G. A. Alvarez, “Number Estimation Relies on a Set of Segmented Objects,” *Cognition*, cxiii, 1 (October 2009): 1–13.

⁴⁰ The idea that adaptation can be diagnostic of perception has been defended by Ned Block, “Seeing-As in the Light of Vision Science,” *Philosophy and Phenomenological Research*, LXXXIX, 3 (November 2014): 560–72. See also Tyler Burge, “Reply to Block: Adaptation and the Upper Border of Perception,” *Philosophy and Phenomenological Research*, LXXXIX, 3 (November 2014): 573–83. Neither Block nor Burge mention numerical perception, but my discussion here is informed by many of the broader points they make.

⁴¹ David Burr and John Ross, “A Visual Sense of Number,” *Current Biology*, xviii, 6 (Mar. 25, 2008): 425–28.

⁴² You can confirm this experience yourself with the following demo (from *ibid.*, supplemental data): <https://ars.els-cdn.com/content/image/1-s2.0-S0960982208002388-mm1.pdf>.

⁴³ M. Fornaciai, G. M. Cicchini, and D. C. Burr, “Adaptation to Number Operates on Perceived Rather Than Physical Numerosity,” *Cognition*, cli (June 2016): 63–67.

⁴⁴ Burr and Ross, “A Visual Sense of Number,” *op. cit.*, p. 426. See my n. 42 for a link to the demonstration.

of a conceptual overlay on perception.”⁴⁵ One *prima facie* reason to think that the adaptation is perceptual is that the phenomenal effect is strong and introspectively seems to be visual. The displays have a strong air of *looking* different following adaptation, which is defeasible evidence that they *do* look different—that is, that they differ in *visual* phenomenology. But we needn’t rely solely on introspection. Controlled studies can also provide evidence that the adaptation is perceptual. For example, researchers have shown that numerical adaptation is spatially selective in external coordinates.⁴⁶ If you look at a large number of dots at one location, you will underestimate the number of dots in a new display, but only if the dots are presented at the same location in external space. While not definitive, this evidence is suggestive of a perceptual effect.

Returning to the issue at hand, we can reframe the two objections to the argument from Weber’s Law in terms of the perception of numerical magnitudes. According to the world objection, a full explanation of why subjects obey Weber’s Law with respect to their numerical discriminations can be pinned on the similarities among numerical magnitudes in the world. Perceivers obey Weber’s Law because (say) ten dots are more similar to nine dots than to eight dots, and so any process that converts the numerical information in the world into a representation in the mind (even a digital one) is more likely to confuse the pair of ten and nine dots than the pair of ten and eight dots. According to the conversion objection, by contrast, a full explanation of Weber’s Law can be blamed on the noisy process by which sensory inputs are converted into digital representations. Eight dots are most often converted into a digital representation of eight dots, but they are also sometimes converted into a digital representation of seven or nine dots, other times converted into a digital representation of six or ten dots, and so on. Weber’s Law thus results.

A study by Robert Moyer and Thomas Landauer tells against both of these objections.⁴⁷ Moyer and Landauer presented numerate adults with two Arabic numerals (such as ‘6’ versus ‘9’) side by side, and asked

⁴⁵ Block, “Seeing-As in the Light of Vision Science,” *op. cit.*, p. 566. Although Block’s formulation here is helpful, it is also misleading in one respect: it implies that all of cognition is conceptual. It is not. As a result, showing that “concepts don’t adapt in the way that percepts do” (*ibid.*) would not suffice to show that adaptation is perceptual. For further discussion, see Burge, “Reply to Block,” *op. cit.*, and Grace Helton, “Recent Issues in High-Level Perception,” *Philosophy Compass*, xi, 12 (December 2016): 851–62.

⁴⁶ Roberto Arrighi, Irene Togoli, and David C. Burr, “A Generalized Sense of Number,” *Proceedings of the Royal Society B: Biological Sciences*, cclxxxi, 1797 (Dec. 22, 2014): <https://doi.org/10.1098/rspb.2014.1791>.

⁴⁷ Robert S. Moyer and Thomas K. Landauer, “Time Required for Judgements of Numerical Inequality,” *Nature*, ccxv (Sep. 30, 1967): 1519–20.

them to indicate the numeral referring to the larger integer by pressing the appropriate button, left or right, as quickly as they could. Note that the subjects were not presented with numerical quantities themselves (such as a grouping of six dots versus a grouping of nine dots). They were presented with digital representations of quantities (that is, Arabic numerals). So if the vehicles that enable numerical discriminations are digital, there is no reason to expect any effects of Weber's Law. Even if the sensory process by which Arabic numerals are converted into internal digital representations is noisy, the Arabic numeral '7' is less physically similar to the Arabic numeral '8' than to the Arabic numeral '1', and so there is no reason to expect greater difficulty discriminating '7' from '8' than '7' from '1'; quite the opposite. And yet, Moyer and Landauer found that the subjects' response times and errors increased as a constant, linear function of the ratio of the two integers. As the ratio between the integers approached 1:1, subjects made proportionately more mistakes and were proportionately slower to tell which of the two numerals referred to the larger number. This effect never went away, even after thousands of trials and even when the subjects were professional mathematicians.⁴⁸ Moreover, when researchers used two-digit numerals, errors and reaction times exhibited comparable profiles whether the comparisons were across decades (for example, 69 versus 71) or within decades (for example, 71 versus 73).⁴⁹ This is surprising since a natural strategy is to first compare the digits in the tens' place of each numeral, and then only compare the digits in the ones' place of each numeral when the former are identical. But that is not what subjects did. Rather, they seemed to treat each two-digit numeral holistically.

These results are hard to explain if Weber's Law is presumed to be a product of similarities among stimuli in the world (the world objection) or a noisy conversion process into digital representations (the conversion objection). But they are exactly what one would expect if perceivers automatically convert their interpretations of the Arabic numerals into analog vehicles and then decide which value is larger by comparing those analog vehicles. Similar findings have been reported for some other symbolically represented magnitudes, including

⁴⁸ Dehaene, *The Number Sense*, *op. cit.*, pp. 62–63.

⁴⁹ James V. Hinrichs, Dale S. Yurko, and Jing-Mei Hu, "Two-Digit Number Comparison: Use of Place Information," *Journal of Experimental Psychology: Human Perception and Performance*, vii, 4 (August 1981): 890–901; and Stanislas Dehaene, Emmanuel Dupoux, and Jacques Mehler, "Is Numerical Comparison Digital? Analogical and Symbolic Effects in Two-Digit Number Comparison," *Journal of Experimental Psychology: Human Perception and Performance*, xvi, 3 (August 1990): 626–41.

length⁵⁰ and area.⁵¹ At least for these magnitudes, we thus have evidence of analog vehicles that are internal to the mind.

One might object that Moyer and Landauer's study cannot be relevant to the question whether *perceptual* vehicles are analog since the vehicles they studied were cognitive, not perceptual. The only things perceived were Arabic numerals, and just as perceiving the word 'red' does not suffice to perceive the color red, perceiving the numeral '7' does not suffice to perceive the number seven. Representations of number only enter later as a cognitive interpretation of the perceived numerals.

But this objection misconstrues the dialectic. Moyer and Landauer's study is not supposed to establish that perceptual vehicles are analog all by itself. It is just supposed to block the conversion objection. For recall that the conversion objection predicts that Weber's Law should not arise in the absence of conversion, and Moyer and Landauer's study shows that it does. (The study also blocks the world objection since there are no numerical magnitudes in the world to confuse.)

Still, Moyer and Landauer's study does not put the conversion objection completely to rest. It remains possible that Weber's Law is explained in cognition by analog vehicles but that Weber's Law in perception is explained by a noisy conversion into digital vehicles. Alternatively, it remains possible that representations of number, length, and area involve analog vehicles in both perception and cognition, but that the representation of other magnitudes (such as luminance) only give rise to Weber's Law because of a noisy conversion into digital vehicles.

While I grant that it is *possible* that Weber's Law results from analog vehicles in some cases and a noisy conversion process in others, I do not think that that is very likely. Weber's Law is idiosyncratic. It makes highly specific quantitative claims. For it to arise, the noise needs to be just so. It would thus be extremely surprising if it emerged in two entirely independent ways in perception and cognition, or in the representation of different magnitude types. It screams out for a unified explanation. Only by positing analog representations across the board can such a unified explanation be provided.

Given that we have such strong reason to posit analog vehicles for certain magnitude types in cognition, there is thus a plausibility

⁵⁰ Tom Trabasso, Christine A. Riley, and Elaine Gumer Wilson, "The Representation of Linear Order and Spatial Strategies in Reasoning: A Developmental Study," in R. Falmagne, ed., *Reasoning: Representation and Process in Children and Adults* (Hillsdale, NJ: Lawrence Erlbaum Associates, 1975), pp. 201–29.

⁵¹ Robert S. Moyer and Richard H. Bayer, "Mental Comparison and the Symbolic Distance Effect," *Cognitive Psychology*, viii, 2 (April 1976): 228–46.

argument to be made that we should also posit them for other magnitude types, and in perception as well as cognition. Still, I admit that this is ultimately an empirical issue, and in the long run we should not be satisfied with plausibility arguments. So, I also want to review a further line of research that shows how empirical work may be able to supply converging evidence.

Andreas Nieder and Earl Miller provide evidence that numerical vehicles have the same format whether they are perceptual or cognitive.⁵² Using a delayed match-to-sample task, they presented rhesus monkeys with a set of dots on a screen (the sample) and then, after a one-second delay, required the monkeys to determine whether a second set of dots (the test) were equinumerous. The appearance of the displays varied to control for low-level features such as area, circumference, density, and shape. While the monkeys performed the behavioral task, the experimenters recorded from individual neurons in the lateral prefrontal cortex during both the *coding phase*, when the samples were presented and thus perceived, and the *delay phase*, when nothing was on the screen but the monkeys were presumably keeping a representation of dot number in working memory to compare to the test.

The monkeys succeeded on the behavioral task in conformity with Weber's Law. With respect to the neural data, Nieder and Miller found that the neurons from which they recorded were tuned to specific numerical values. For example, a "three-neuron" fired most when the display contained three dots, less when it contained two or four dots, and so on. They also found that these bell-shaped firing patterns exhibited scalar variability, just as one would predict if they were the vehicles responsible for scalar variability at the behavioral level. Moreover—and here is the kicker—the tuning functions did not change between the coding and delay phases, suggesting that perceptual and post-perceptual vehicles of numerical representation "share the same fundamental mechanisms and neural coding schemes," and thus the same format.⁵³

⁵² Andreas Nieder and Earl K. Miller, "Coding of Cognitive Magnitude: Compressed Scaling of Numerical Information in the Primate Prefrontal Cortex," *Neuron*, xxxvii, 1 (Jan. 9, 2003): 149–57.

⁵³ *Ibid.*, p. 149. The appeal to "three-neurons" and the like should not mislead one into accepting an oversimplified picture of how numerical information is coded. For example, one should not conclude that a subject represents three entities just in case a single three-neuron fires above some threshold. Single-neuron responses are noisy, and the results reported by Nieder and Miller are averaged over many trials. To get an accurate numerical estimate on a single trial, the brain thus presumably consults populations of neurons. Exactly how this works remains an open question, though one possibility, consistent with the idea that the brain uses magnitudes to represent magnitudes, is that it

Nieder and Miller tested monkeys, not humans, and so one might wonder whether their results would extrapolate. But the mechanisms underlying “the number sense” (as Dehaene⁵⁴ calls it) are widely believed to be evolutionarily ancient, so it is likely that similar results would obtain in humans were it possible to undertake single-cell recordings with humans in the same way. (The procedure is invasive, so for ethical reasons it usually is not.) A more serious concern about this study is that the delay phase is only one second long, and so tells us little about the format that numerical representations might take when they are stored in long-term memory, which is presumably where the representations that are triggered by Arabic numerals in Moyer and Landauer’s study come from. It would thus be nice to see a version of the study that involved a longer delay period or (better) used numerals as inputs in symbolically trained monkeys. And, of course, it would also be nice to see versions of the study that target a wide variety of non-numerical magnitudes. At the very least, however, the study illustrates how empirical findings can speak to the issue.

If Weber’s Law is a contingent property generated by the mind, it is natural to wonder why it would arise. Why would minds have evolved to exhibit such ratio sensitivity? This is a challenging question at the foundations of psychophysics, but one speculative suggestion is that ratio sensitivity is a consequence of two constraints: limited representational resources in the mind, and a decreasing need by organisms to discriminate among objective magnitudes as their values increase. Given the former constraint, the mind cannot be expected to represent objective magnitudes with infinite precision. Given the latter constraint, representational precision should decrease as the objective magnitude increases. Something approximating Weber’s Law might thus be expected to result.⁵⁵

uses a population rate code. See Andreas Nieder, “The Neuronal Code for Number,” *Nature Reviews: Neuroscience*, xvii (June 2016): 366–82.

⁵⁴ Dehaene, *The Number Sense*, *op. cit.*

⁵⁵ See Steven T. Piantadosi, “A Rational Analysis of the Approximate Number System,” *Psychonomic Bulletin and Review*, xxiii, 3 (June 2016): 877–86, for an analysis along these lines and a helpful discussion of some previous analyses. Piantadosi argues that the assumption that lower magnitude values have a greater “need probability” than higher magnitude values receives empirical support in the case of number from the frequency with which individual number words are used (as measured, for example, by the Google Books N-gram dataset). This evidence is intriguing but not decisive. Frequency of use may not track need. More generally, I take it to be an open question, deserving of further exploration, whether Weber’s Law can really be rationalized in this way.

VII. SENSORY MAGNITUDES

So far, I have been arguing that perceptual vehicles are analog without considering how those vehicles relate to experience. Two possibilities should be distinguished. According to the first, perception has analog vehicles, but those vehicles are purely subpersonal. They cannot be located in experience itself. For example, they might be exhaustively characterized in terms of some neural magnitude, such as neural firing rate. According to the second possibility, perception has analog vehicles, and those vehicles are, in fact, reflected in experience, at least when perception is conscious. Those vehicles might *also* have a characterization in other—say, neural—terms. It is just that they are not *merely* characterizable in (say) neural terms. They are simultaneously characterizable in the vocabulary of experience. In this section I want to tentatively explore this second possibility. My aim is not to settle the issue, but to show how it is intimately linked to some controversies in psychophysics.

When you represent objective magnitudes in perception, there is something it is like for you to experience those magnitudes. We can capture this phenomenal aspect of perception by talking about *sensory magnitudes* that correspond to the objective magnitudes you represent. For example, luminance (an objective magnitude) corresponds to brightness (a sensory magnitude), sound-wave amplitude corresponds to loudness, lifted weight corresponds to felt effort, temperature corresponds to warmth or coldness, sucrose concentration corresponds to sweetness, citric-acid concentration corresponds to sourness, and so on. The justification for calling these phenomenal aspects of perception *magnitudes* is that they can be quantitatively compared along a dimension. Experiences can be more or less bright, more or less sweet, and so on.

Two brief comments about sensory magnitudes are in order. First, using the terms 'loudness', 'brightness', 'sweetness', and so on to refer to sensory magnitudes may raise the eyebrows of philosophers who prefer to reserve these terms for the complex non-objective magnitudes that perception tracks more closely than objective magnitudes (for example, such that 'loudness' refers to the physical magnitude that is a complex function of sound-wave amplitude, frequency, bandwidth, and duration). Such philosophers may prefer to use the terms 'phenomenal loudness', 'phenomenal brightness', 'phenomenal sweetness', and so on to refer to sensory magnitudes. But I take this to be a terminological matter. Those who prefer to do so can append the term 'phenomenal' to my use of the terms 'loudness', 'brightness', and so on.

Second, there are, rather notoriously, a variety of views one could adopt about the natures of sensory magnitudes. According to representationalists or intentionalists, their natures can be reductively explained in terms of the representation of (objective or non-objective) magnitudes. According to functionalists, their natures can be reductively explained in terms of their functional roles. According to identity theorists, their natures can be reductively explained in terms of their neural or other physical properties. According to qualia theorists, their natures cannot be reductively explained at all. For present purposes, we can remain neutral with respect to these views.

Weber's Law is indicative of analog vehicles, but it does not tell us what those vehicles are. The recognition of sensory magnitudes generates an obvious suggestion. Perhaps the analog vehicles should be identified with sensory magnitudes. If so, the analog vehicles are not merely subpersonal, but are located in sensory experience itself. To investigate the plausibility of this suggestion we can look to psychophysics for a description of the relation between sensory magnitudes and objective magnitudes.

One description of that relation was provided by Gustav Fechner.⁵⁶ According to what has come to be known as *Fechner's Law*: $S = k \log(M)$, where S is the intensity of the sensory magnitude (such as brightness), k is a constant, and M is the intensity of the objective magnitude (such as luminance). Fechner derived this law from Weber's Law together with the assumption that each just-noticeable difference of the objective magnitude corresponds to an equal sensory change—for example, that each just-noticeable difference in luminance corresponds to the same change in brightness. The resulting logarithmic relation means that a set change in M has larger effects on S at lower values of M than at higher values of M . Thus, adding one candle to one candle has a larger effect on brightness than adding one candle to two candles, which has a larger effect on brightness than adding one candle to three candles, and so on.

But Fechner's Law (and the equal-sensory-change assumption that generates it) has been challenged, most prominently by Stanley Stevens.⁵⁷ Stevens presented subjects with a stimulus and gave it an arbitrary number (say, 10) or allowed subjects to assign an arbitrary number to it. He then asked subjects to assign further numbers to new

⁵⁶ Gustav T. Fechner, *Elemente der Psychophysik* (Leipzig: Breitkopf und Härtel, 1860).

⁵⁷ S. S. Stevens, "To Honor Fechner and Repeal His Law," *Science*, cxxxiii, 3446 (Jan. 13, 1961): 80–86; and S. S. Stevens, *Psychophysics: Introduction to Its Perceptual, Neural, and Social Prospects* (New York: Wiley, 1975).

stimuli such that their sensed intensity would be proportional to the original assignment (for example, if the original stimulus received a 10, a new stimulus with double the sensed intensity should receive a 20). Stevens found that subjects' assignments of numbers were surprisingly consistent. He captured the pattern of their responses by what is now known as *Stevens's Power Law*: $S = kM^b$, where S and M are as in Fechner's Law, k is a constant (different from the constant in Fechner's Law), and b is an exponent that is characteristic of a given magnitude dimension. For some objective magnitudes (such as luminance) $b < 1$, roughly along the lines of Fechner's Law. But for others $b > 1$ (for example, lifted weight) or $b = 1$ (for example, visually presented length).

Recall that explanations of Weber's Law in terms of analog representations have two components: a mapping from objective magnitudes to the analog vehicles that represent those magnitudes, and an accompanying pattern of noise. Fechner's Law and Stevens's Law offer disparate monotonic mappings from objective magnitudes to sensory magnitudes. On the assumption that sensory magnitudes are the analog vehicles that represent objective magnitudes, Fechner's Law and Stevens's Law thus become competing hypotheses about the first component in an explanation of Weber's Law. But that still leaves the second component, the accompanying pattern of noise, about which Fechner's Law and Stevens's Law are both silent. To explain Weber's Law, we would thus need to make further assumptions about the second component—the accompanying pattern of noise.

For Fechner's Law, the assumption we would need to make is simple enough. When the mental magnitude is a logarithmic function of the objective magnitude, constant noise will engender Weber's Law. If Fechner's Law is correct, sensory magnitudes thus make excellent candidates to serve as the analog vehicles that Weber's Law gave us reason to posit. We need only assume that sensory magnitudes are beset by a constant amount of noise, which is hardly farfetched.

But suppose that Stevens's Law is correct. In order for sensory magnitudes to serve as our analog vehicles, we would then need to posit a variety of different patterns of noise that just so happen to complement the disparately structured sensory magnitudes in a way that yields Weber's Law. For example, the noise would need to be roughly constant for some magnitudes (such as luminance), increase linearly in proportion to the magnitude value for others (such as visually presented length), and increase exponentially for still others (such as lifted weight). While not incoherent, this hypothesis lacks the

prima facie plausibility that accompanies the hypothesis that Weber's Law results because all sensory magnitudes have the same logarithmic structure and are accompanied by a constant level of noise. The idea that the vehicles of conscious perception consist in sensory magnitudes thus fits much more naturally with Fechner's Law than with Stevens's Law.

Note that the problem I am raising is not that it is implausible that different sensory magnitudes might be accompanied by different patterns of noise. Different sensory magnitudes are served by different transducers and different neural populations, so different kinds of noise might be expected. Rather, what is implausible is that the different patterns of noise would be *exactly such* that they combine with the differently structured sensory magnitudes to yield Weber's Law. For recall that each differently structured sensory magnitude has to be complemented by a very specific pattern of noise if it is to generate Weber's Law (constant noise for a logarithmic structure, scalar noise for a linear structure, and so on). So if sensory magnitudes are the analog vehicles of perceptual discrimination and Stevens is right that sensory magnitudes have a variety of different monotonic structures, each of those structures would not only need to be accompanied by a different type of noise, but by a *very specific* type of noise. And short of some cosmic coincidence or pre-established harmony, it is hard to imagine why those very specific patterns of noise would emerge.

Fechner's Law and Stevens's Law are both controversial, and this is not the place to choose between them. I introduce them only to show how the question whether analog vehicles can be located in conscious perception is intimately connected to controversies within psychophysics about how sensory magnitudes are related to objective magnitudes.

VIII. CONCLUSION

In the 1980s, philosophers advanced a number of arguments that perception is analog. These arguments were then extensively criticized, and philosophical discussions of the issue largely petered out. I have tried to show how we can make fresh progress by developing the argument from Weber's Law. To wrap up, I want to briefly contrast the earlier arguments with the argument from Weber's Law.

One significant difference concerns the conceptions of analog representation presupposed by each argument. Dretske has his own idiosyncratic conception (see note 23). The argument I associated with Evans presupposes the continuous conception. Peacocke's appeal to experience's fine-grained character is also suggestive of the continuous conception.⁵⁸ The argument from Weber's Law, by contrast, relies on a covariational version of the mirroring conception, and thus has a subtly different conclusion than the earlier arguments.

The arguments also differ in *how* they support their conclusions. The earlier arguments all use facts about the *content* of perception—that color, shape, and size are represented all at once (Dretske), that fine-grained color shades are represented (Evans), or that magnitudes are represented without units (Peacocke)—to draw conclusions about the *format* of perception. As a result, these arguments are open to two challenges. First, the inference from content to format can be challenged on the grounds that the same objects and properties can generally be represented in multiple formats. The directions between two locations can be represented by a map or a list of sentences. The percentage of the population that voted in the last election can be represented with a pie chart or a histogram. Likewise, fine-grained color shades can be represented by analog or digital representations even when they are bundled with information about shape and size. Second, one can challenge the arguments' claims about which properties are represented in perception. In particular, Peacocke's claim that perception does not traffic in units is open to question. The argument from Weber's Law, by contrast, is immune to both sorts of challenges because it derives conclusions about format not from perception's content, but from the patterns of error associated with perceptual discrimination. In this respect, it is reminiscent of arguments from cognitive science that mental imagery is pictorial.⁵⁹ I believe that

⁵⁸ However, Peacocke's claim that experience is unit free together with his explicit discussion of Shepard's concept of a second-order isomorphism are suggestive of the mirroring conception. So perhaps he should be interpreted as operating with a hybrid conception in his early work. See Peacocke, "Analogue Content," *op. cit.*, pp. 6–8; and Peacocke, "Perceptual Content," *op. cit.*, pp. 304–06. In more recent work, Peacocke disavows the idea that analog representations must be continuous and develops an account of analog content in terms of recognizability. See Peacocke, "Magnitudes," *op. cit.*

⁵⁹ See Roger N. Shepard and Jacqueline Metzler, "Mental Rotation of Three-Dimensional Objects," *Science*, CLXXI, 3972 (Feb. 19, 1971): 701–03; and Kosslyn, *Image and Mind*, *op. cit.* There is, however, at least one respect in which the argument from Weber's Law may be better off. Arguments that mental imagery is pictorial always struggled to specify the respects in which imagery representation needs to be picture-like to count as *pictorial*. By contrast, the sense of *analog* presupposed by the argument from

places the argument on a firmer foundation. At the very least, that places it on a different foundation.

The upshot is that we have a new argument that perception is analog that improves upon earlier efforts. An outstanding question, only likely to be answered in collaboration with psychophysics, is whether the analog vehicles that this new argument provides reason to posit can be located in sensory experience.

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