Analogue Magnitudes, the Generality Constraint, and Nonconceptual Thought

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I reply to comments by David Miguel Gray and Grant Gillett concerning my paper, 'The Generality Constraint and the Structure of Thought'.

1. The main argument revisited

In Beck 2012 I argued as follows.

(P1) Mental states with conceptual content must be systematic

(P2) Analogue magnitude states have content but are not systematic

Thus,

(C) Analogue magnitude states have nonconceptual content

(P1) is what Evans (1982) calls the *Generality Constraint*. It holds that mental states with conceptual content must be closed under all meaningful recombinations of the constituents of the sentences that best express them. For example, anyone who can conceptually think that Albert is friendly and that Bob is gracious must also be able to conceptually think that Albert is gracious and that Bob is friendly. The Generality Constraint does not deny that some mental states might fail to be systematic. It just maintains that non-systematic mental states will lack conceptual content. Gillett thus seems to misunderstand my argument when he claims that I 'argue that the [generality] constraint is not definitive of conceptual states in the way that Evans claimed' (Gillet 2014, p. 1147). On the contrary, I assume that the Generality Constraint is a necessary truth and use it as a premiss in my argument for (C). At no point do I deny that the Generality Constraint is definitive of conceptual content.

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The most substantial portion of Beck 2012 was committed to defending (P2). For the sake of concreteness, I focused on a specific experiment that taps into the analogue magnitude states of pigeons. Rilling and McDiarmid (1965) trained pigeons to repeatedly peck a centre, illuminated key until it extinguished. If the experimenter extinguished the centre key after exactly 50 pecks, the pigeon was rewarded for pecking the right-most key. If the experimenter extinguished the centre key after n pecks, such that $n < 50$, the pigeon was rewarded for pecking the left-most key. Pigeons succeeded ninety percent of the time when n was 35, seventy percent of the time when n was 43, and fifty percent of the time (chance) when n was 47.

I emphasized two features of this experiment. First, the behaviour of pigeons conforms to Weber's Law, which holds that the ability to discriminate two magnitudes decreases as their ratio approaches one. Beyond a certain ratio (9:10 for pigeons), discrimination falls apart. Second, the fact that pigeons generally succeed at this task (albeit within the limits imposed by Weber's Law) suggests that they are able to represent some number or number-like property.

I used this experiment to construct an argument for (P2). To a first approximation, the argument maintains that pigeons can represent that:

(1) 40 pecks are fewer than 50 pecks

and that

(2) 38 pecks are fewer than 47 pecks

but not that

(3) 38 pecks are fewer than 40 pecks

nor that

(4) 47 pecks are fewer than 50 pecks

Thus, pigeons' analogue magnitude states are not systematic. They are not closed under all meaningful recombinations of the constituents of the sentences that best express them. It follows in conjunction with (P1) that pigeons' analogue magnitude states have nonconceptual content. Moreover, because humans have analogue magnitude states too, the argument can be extended to us (Beck 2012, pp. 584–5). We humans also have cognitive states with nonconceptual content.

As I noted in Beck 2012, there are many ways to object to this argument, some of which I am sympathetic towards. For example, one might object that there are no sentences that can precisely characterize the contents of analogue magnitude states since those states have nonconceptual content and sentences can only express conceptual content. I agree. However, this objection has no force in blocking my argument for (C) since it presupposes (C). The objection brings out only that my argument has the form of a *reductio ad absurdum*. My argument assumes, for the sake of *reductio*, that analogue magnitude states have conceptual content and can thus be expressed using sentences. I emphasize this point because Gillett seems to think it obvious that pigeons' mental states are not conceptual in anything like the sense that Evans (1982) isolated. He writes, 'it seems rash in the extreme to align the pigeons' abilities with the kind of thing the generality constraint is trying to elucidate' (Gillet 2014, p. 1149). I am glad to have him as an ally, though I am less confident that the point is obvious. It has certainly been denied (Fodor 1987, Fodor and Pylyshyn 1988, Carruthers 2004 and 2009). In any case, Gillett and I can both agree that (C) is true even if we get there by different routes.

Another objection that I am sympathetic towards maintains that (1)–(4) are not the best sentences we can find to express the contents of analogue magnitude states. As I pointed out (Beck 2012, pp. 579–80), there are reasons to be unhappy with the attribution of precise integer concepts such as *47* and *50* to pigeons. A better attempt to express the contents of pigeons' analogue magnitude states would appeal to approximate concepts such as *47ish* and *50ish*. I argued, however, that the move to approximate concepts does not save systematicity (Beck 2012, pp. 580–2). Gray (2013) challenges this argument of mine.

2. Reply to Gray

Gray distinguishes two interpretations of approximate numerical concepts such as *38ish* and *50ish*. According to what he calls the 'centred account', these concepts are 'centred around numerical concepts' — for example, '38ish is centred on 38' (Gray 2014, p. 1144). According to the 'diffuse account', by contrast,

> the numerical concept is not taken to be part of the numerical-ish concept itself. In other words, while we may describe 38ish as '38 more or less' on

> both accounts, on the diffuse account, 38 does not have a centred or privileged position over above any other number that falls within the domain of 38ish. (Gray 2014, p. 1144)

Gray does not elaborate on what it means for 38 to have a 'centred' or 'privileged' position, but he is clear about one consequence he sees for the diffuse account. According to the diffuse account,

(1*) 40ish pecks are fewer than 50ish pecks

and

(2*) 38ish pecks are fewer than 47ish pecks

are both true. But

(3*) 38ish pecks are fewer than 40ish pecks

and

(4*) 47ish pecks are fewer than 50ish pecks

are both false. Gray concludes, 'Thus, numerical-ish concepts can recombine and the Generality Constraint is preserved' (Gray 2014, p. 1145).

I believe that Gray is guilty of a non sequitur. Even if the diffuse account renders (3*) and (4*) false, it does not follow that pigeons can *represent* (3*) and (4*), which is what the Generality Constraint requires. Moreover, Gray overlooks evidence that pigeons *cannot* represent (3*) and (4*). Before I develop this reply, however, I want to clarify what is at issue by distinguishing some claims that are in the vicinity of the diffuse account.

One claim, suggested by Gray's comment that 'the numerical concept is not taken to be part of the numerical-ish concept itself', is that concepts such as *38ish* should not be viewed as structured from two components, the concept *38* and the concept *ish*. I agree. My reason for introducing concepts such as *38ish* was to accommodate the fact that pigeons cannot represent precise integers such as 38. Obviously, if pigeons lack the concept *38* they must lack complex concepts that contain *38* as a component. Thus, whatever differences are supposed to exist between Gray's diffuse account and the centred account that he attributes to me, this cannot be one of them.

A second claim in the vicinity of the diffuse account is that a concept such as *38ish* represents a range of values (say, from 35 to 42) without in any way privileging 38 over any other value in that range. Gray seems to endorse this claim when he writes, 'on the diffuse

account, 38 does not have a centred or privileged position over above any other number that falls within the domain of 38ish' (Gray 2014, p. 1144). The problem with this claim, however, is that pigeons' analogue magnitude states *do* privilege certain numbers over others. If you train a pigeon to peck a key 38 times, the pigeon's responses will form a bell curve around 38, with exactly 38 being the mean and mode response. Presumably Gray does not mean to deny that *38ish* privileges 38 in this sense.

A third claim in the neighbourhood is that when two analogue magnitude concepts are related by a comparative such as *fewer than*, the resulting content is false when the ratio of the two concepts exceeds the subject's Weber ratio, as in (3*) and (4*). Gray embraces this claim when he writes,

> the diffuse account would rule that (3*) is false: 38ish pecks are not fewer than 40ish pecks. Given that the numbers involved in the descriptions are in a ratio that exceeds 9:10 a contrastive relationship cannot be drawn. (Gray 2014, p. 1144).

While it is not clear to me what 'diffuseness' is supposed to be such that this third claim follows, I propose to waive that worry.[1] For the sake of argument, I will grant that this third claim follows from the diffuse account. Even so, I will argue that Gray has not managed to save the systematicity of analogue magnitude states.[2]

The Generality Constraint lays down a recombinability condition on the contents one must be capable of representing that applies even when the recombinations yield contents that are false. For example, if you can conceptually think that $1+1=2$ and that $2+2=4$, the Generality Constraint requires that you be able to think that $1+1=4$. Of course, you need not *believe* that $1+1=4$. But you do need to be able to represent the proposition — if only to represent it as false. Similarly, if a pigeon can conceptually represent (1*) and (2*), the Generality Constraint requires that the pigeon be able to represent

[1] Whatever diffuseness is supposed to be, the concepts *many* and *slightly-more-than-many* are presumably diffuse. Nonetheless, many pecks are fewer than slightly-more-than-many pecks. So diffuseness does not seem to suffice to make fine-grained comparisons false. It is thus unclear to me why Gray takes the diffuseness of analogue magnitude concepts to entail that (3*) and (4*) are false.

[2] In Beck 2012, I consider and reply to two interpretations of analogue magnitude states that maintain that (3*) and (4*) are false: alternative #4 (p. 579) and alternative #6 (pp. 580–1). Because Gray says little about what 'diffuseness' is, I am uncertain how his interpretation is meant to differ from these alternatives. In any case, my reply to Gray will develop and expand upon my earlier replies.

(3*) and (4*) as well — if only to represent that they are false. To demonstrate that pigeons obey the Generality Constraint, it is thus not enough for Gray to show that (3*) and (4*) are false. He must also show that pigeons can represent (3*) and (4*).

All else equal, if an organism can represent a proposition, it should be possible to find evidence of that fact. Thus, we attribute the ability to represent (1*) to pigeons in part because they demonstrate the ability to discriminate 40 pecks from 50 pecks. By contrast, we do not attribute the ability to represent that the Cold War was the defining struggle of post-World War II Europe to pigeons because there is no evidence, manifested in their behaviour or elsewhere, that they can represent that proposition. Thus, all else equal, if pigeons can represent (3*) and (4*) it should be possible to find evidence of that fact.[3]

Gray points to no evidence that pigeons can represent (3*) and (4*), even as false. Moreover, he overlooks evidence that they cannot. If pigeons could represent (3*) and (4*), then, with all else equal, it should be possible to train a pigeon to peck the left key in response to a ratio that is less than or equal to 9:10 (indicating the truth of claims such as (1*) and (2*)), and the right key in response to a ratio that is greater than 9:10 (indicating the falsity of claims such as (3*) and (4*)). Given a target of 50 pecks, it should thus be possible to train a pigeon to reliably peck the left key when it has pecked 45 or fewer times (indicating the ability to represent (1*) as true), and the right key when it has pecked more than 45 times (indicating the ability to represent (4*) as false). But no one has been able to train a pigeon to do that, and for good reason. Because of Weber's Law, pigeons cannot discriminate 45 from its close neighbours. Thus, pigeons cannot tell whether they have pecked more or less than 45 times. Consequently, they cannot discriminate precise ratios such as 9:10 either. So you cannot train a pigeon to peck one key in response to a ratio that is less than or equal to 9:10, and a second key in response to a ratio that is greater than 9:10. Pigeons do not know what ratios they can and cannot discriminate.

Recall that I introduced the '-ish' suffix in order to accommodate the fact that pigeons cannot represent integers. They represent 50ish, but not 50. Gray is in effect claiming that the '-ish' suffix should be

[3] The 'all else equal' clause is crucial since an organism can have a representational competence that is not manifested in performance. I discuss this point at length in Beck 2012 (pp. 572–6), where I argue that the limits imposed by Weber's Law derive from representational competence. Gray does not challenge this aspect of my argument, so I do not discuss it here.

understood in terms of a precise ratio — for pigeons, 9:10. This comes out in the truth conditions he attributes to analogue magnitude states. Gray maintains that a pigeon's belief that it has pecked fewer than *n*ish times is true just in case it has pecked m times and $m{:}n \leq 9{:}10$. But what Gray seems to be missing is that if pigeons cannot represent integers, they cannot represent ratios among integers either.[4]

To put the point another way, if Gray is right that (3*) is false, and if (as the Generality Constraint requires) pigeons can represent (3*), then all else equal when a pigeon is asked whether (3*) is true, there should be some circumstance under which the pigeon can reliably answer 'No'. But so far as researchers have found, there are no such circumstances. And given Weber's Law, there is reason to think that no such circumstances will ever be found. Weber's Law suggests that pigeons are incapable of grasping propositions such as (3*), not that they believe such propositions to be false.

This problem applies to Gray's alternative suggestion that pigeons operate with the concept *fewerish* such that x is fewerish than y 'if and only if x is less than y and the ratio between x and y is less than 9:10' (Gray 2014, p. 1145). If the pigeons were operating with such a concept, then they should evince the ability to represent a ratio of exactly 9:10 — for example, by pecking the right key just in case $n > 45$ when the target is 50. But they evince no such ability. Thus, systematicity is not saved.

Gray's account misplaces the significance of Weber's Law. Weber's Law describes the discriminative abilities of pigeons. It describes what magnitudes pigeons can and cannot discriminate. But Weber's Law does not describe the *contents* of pigeons' analogue magnitude states. Pigeons do not represent what magnitudes they can and cannot discriminate, and hence do not represent whether a ratio is greater or less than 9:10. Pigeons are not that clever.

3. Reply to Gillett

Gillett (2014) structures his remarks around three 'observations'. I will discuss each in turn.

[4] Notice that it would not help Gray to insist that pigeons represent ratios among integers, but not *as* ratios among integers (i.e. that they represent ratios among integers *de re* rather than *de dicto*). The inability of pigeons to discriminate among precise ratios in any circumstance suggests that there is *no* mode of presentation under which they represent ratios among integers.

According to the first, 'Discriminatory capacities are not conceptual states' (Gillet 2014, p. 1148). I agree, and never claimed otherwise. Conceptual states must satisfy certain conditions, including the Generality Constraint. Pigeons and other animals can discriminate magnitudes, but they do not satisfy the Generality Constraint. Thus, their magnitude discriminations are not grounded in conceptual states.

There are surely other requirements, apart from the Generality Constraint, that must be met for conceptual thought. Gillett includes the ability to 'conceive of an objective world' and to refer to that world 'in thinking and speaking' (Gillet 2014, p. 1149). While I disagree that conceptual thought requires speech, nothing in my argument for (C) requires me to take a stand on the issue. I argued that pigeons and other organisms violate systematicity with respect to their analogue magnitude states. I did not claim that they satisfy (or violate) other requirements that might be placed on conceptual thought.

Gillett's second observation is that the sorts of mental representations underlying Evans's discussion of the Generality Constraint are different from those I attribute to pigeons. Again, I agree. The representations I attribute to pigeons have an analogue structure; the representations underlying Evans's discussion have a discrete, predicative structure. Gillett suggests that a theory of conceptual thought such as Evans's (1982) need not be accompanied by a syntactic theory of mental representation such as Fodor's (1987). This is true; it *need* not. It is logically possible to follow Evans (1982, pp. 100–1) and abstain from any commitment to a language of thought. I argued, however, that if we are to take seriously the contention that Weber's Law is best explained by representations with an analogue format, we are pressured to also take seriously the claim that the Generality Constraint is best explained by representations with a predicative format (Beck 2012, pp. 589–94). In each case, the inference is non-demonstrative, but that does not mean that it is lacking in force. Gillett neither challenges the inference from Weber's Law to an analogue format, nor explains why that inference should be more acceptable than the inference from the Generality Constraint to a predicative format.

Gillett sometimes hints that he rejects the attribution of *any* contentful mental states to pigeons. He invokes behaviourist terms such as 'operant response' to describe pigeon behaviour and alludes to 'more parsimonious' non-cognitive explanations 'such as the ability of an avian nervous system to approximate, within the parameters of

the appropriate instance of Weber's law, a chained pecking response previously emitted and meeting a fixed criterion of success' (Gillet 2014, p. 1152). If this is Gillett's view, however, then it is empirically problematic. Researchers appeal to analogue magnitude representations to explain the numerical behaviour of pigeons and other animals because simpler non-representational explanations, including behaviourist explanations, have thus far proved inadequate (Gallistel 1990; Gallistel and Gibbon 2002; Dehaene 2011; Beck 2012, p. 587, n. 27; Beck forthcoming). Moreover, Gillett's own 'more parsimonious' suggestion is underspecified. Everyone can grant that the avian nervous system approximates 'a chained pecking response', but the question is how it manages to do that. All researchers who have seriously looked into this question have found it necessary to appeal to mental states with numerical content.

Gillett's final observation is that Evans' theory of conceptual thought is aimed at humans, not pigeons, and that humans have many abilities that pigeons lack, including natural language and diverse powers of abstraction that accompany conscious thought. Gillett concludes,

> The point of Evans's Generality Constraint is that it is an attribution forced upon us by an adequate account of what we call conscious thought in a free-ranging world through which we move and, in that respect, [is] unlike such lesser cognitive attainments. (Gillett 2014, p. 1152)

I agree with Gillett that Evans was focused on human thought, not pigeon thought. But so what? If pigeons have cognitive states that represent numerical information, the question whether those states have conceptual content arises. While it may strike Gillett, or me — or even Evans — as 'rash in the extreme' (Gillet 2014, p. 1149) to attribute conceptual thoughts to pigeons, recall that there are those who disagree (Fodor 1987; Fodor and Pylyshyn 1988; Carruthers 2004 and 2009). To advance the discussion, arguments are needed.

There is a further problem with acknowledging that Evans was concerned with conscious human thought, and leaving matters at that. As I emphasized in Beck 2012, analogue magnitude states are not only present in pigeons; they are also present in us. Thus, when Barth et al. (2003) flashed two displays of dots, each for just a second so that there was not enough time to count them, human adults were able to reliably tell which display had more dots so long as the ratio of dots on the two displays was no more than 7:8. Like pigeons, human adults have analogue magnitude states that are governed by Weber's

Law. Moreover, these analogue magnitude states seem to have many of the properties that Gillett is keen to emphasize as constitutive of thought. They are conscious: one display phenomenally seems to have more dots than the other. They concern the objective world. They are implicated in language, such as the use of the term 'more' (Pietroski et al. 2009). They are deployed in natural settings by 'free-ranging' humans, as when you approximate the number of people in a room, the distance to your office, or the time you have spent reading this paper. They are also post-perceptual (Beck 2012, pp. 585–9). Analogue magnitude states are thus reasonably regarded as a type of human thought, even by the criteria Gillett invokes.

The tone of Gillett's discussion suggests that he thinks it is simply absurd to discuss the mental states of pigeons in the context of Evans's Generality Constraint. Yet by ignoring the fact that humans also have analogue magnitude states Gillett misses much of the point of my paper. If analogue magnitude states violate the Generality Constraint and thus have nonconceptual content, that tells us something about ourselves. We humans are not only conceptual thinkers; we are nonconceptual thinkers as well.

Like Gillett, I admire many aspects of Evans's account of conceptual thought. But at the same time, I believe that Evans's account is guilty of a crucial omission: it fails to recognize the existence of nonconceptual thought. This omission is important both because it obscures how heterogeneous human thought is, and because it conceals significant commonalities between human thought and animal thought. While it is important to acknowledge that humans are conceptual thinkers and that pigeons are not, it is also important to recognize that there are forms of nonconceptual thought that humans and pigeons share.[5]

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