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OVERVIEW

Analog mental representation

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Over the past 50 years, philosophers and psychologists have perennially argued for the existence of analog mental representations of one type or another. This study critically reviews a number of these arguments as they pertain to three different types of mental representation: perceptual representations, imagery representations, and numerosity representations. Along the way, careful consideration is given to the meaning of "analog" presupposed by these arguments for analog mental representation, and to open avenues for future research.

This article is categorized under:
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analog, imagery, mental representation, numerosity, perception

1 | INTRODUCTION

A governing idea of cognitive science is that the mind represents the world. Given this governing idea, we can inquire into the format of the mind's representations. This study concerns whether any mental representations have an analog format.

Why should we care whether mental representations are analog? Two general reasons stand out. First, for anyone interested in engineering a mind, the selection of a particular representational format is a fundamental early branch that influences much of what comes later. Thus, for the purposes of reverse engineering actual minds, getting the format of mental representations right will assist in discovering many of the representations' other properties, including the computations they enter into and the contents they possess. Whether minds are analog will thus influence our understanding of what they represent and how they compute.

Second, philosophers have often hypothesized that there is a fundamental cleavage between different kinds of mental states—for example, between perception and thought, or between human cognition and nonhuman animal cognition. These hypotheses, in turn, are often elucidated in terms of differences in representational format. Knowing which mental states are analog should thus be useful for formulating and evaluating hypotheses about how important kinds of mental states differ from one another.

Few researchers think that all mental representations are analog. Rather, discussions of analog mental representation tend to cluster around a small number of candidates. My discussion will focus on three of these: perceptual representations (Section 2), imagery representations (Section 3), and numerosity representations (Section 4). In each case, I will survey some of the most prominent arguments that philosophers and psychologists have marshaled in favor of each type of representation being analog, as well as some difficulties that these arguments face.

It would be nice to commence with a definition of "analog" to serve the forthcoming discussion. But the meaning of "analog" is partly what is at issue, and so definitions will have to be considered along the way. I think it will be helpful, however, to foreshadow two broad conceptions of analog representation under which most (but not all) definitions fall. According to

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the *continuous conception*, analog representations are continuous or dense, or nearly so, rather than discrete or differentiated (Goodman, 1968; Haugeland, 1981; Schonbein, 2014). For example, a mercury thermometer represents the temperature by way of the height of a bead of mercury, which can vary continuously. According to the *mirroring conception*, by contrast, a representation is analog in virtue of being structurally isomorphic to, or otherwise mirroring, what it represents (Blachowicz, 1997; Kulvicki, 2015; Lewis, 1971; Maley, 2011; Shepard & Chipman, 1970). The mercury thermometer is analog according to this conception too since the mercury's height mirrors the temperature.

Perhaps because many paradigmatically analog representations answer to both conceptions, the two conceptions are sometimes conflated—a source of trouble as we will see below. But as Lewis (1971) in effect noted many years ago, they can come apart. Representations that are analog in the mirroring sense need not be continuous. For example, the hands of many traditional clocks advance in discrete steps rather than continuously, but their angle mirrors the time of day.

The concept of analog representation is often contrasted with that of digital representation, but there is arguably more to being digital than *not* being analog—at least if digital representation is supposed to be intimately connected to digital computation. If so, then we should neither expect an account of digital representation to immediately fall out of an account of analog representation, and nor should we expect a delineation of digital mental representations to immediately fall out of an investigation into analog mental representations. In any case, I will have little to say about digital representation in this study. My focus will rest squarely on analog mental representation.

2 | PERCEPTION

2.1 | Evans and Peacocke

Evans (1982) famously observed that our visual experiences of color are fine-grained. We can visually experience many shades of red—far more than we can characterize in terms of our standing color concepts such as scarlet and crimson. Peacocke (1986, 1989, 1992) extended Evans' observation to other perceptible magnitudes, such as distance, size, and orientation. For any given perceptible magnitude, there are indefinitely many values of that magnitude that can be discriminated in perception. This might seem to suggest that the perception of magnitudes is analog in the sense of being continuous. But on reflection, it should be clear that any such inference would be shaky since representation by way of a continuous variable can always be approximated by a discrete representation up to an arbitrarily fine degree of precision. Think, for example, of a digitized picture whose pixels are so small that they are indiscernible to the naked eye. The mere fact that there are lots of colors (distances, sizes, etc.) that we can perceptibly discriminate thus does not mean that colors are represented by a continuous vehicle.

But when Peacocke argues that perceptual experience is analog, he does not only rely on the observation that it is fine-grained. He adds a second observation into the mix.

> Consider someone who has to furnish a room. He may see how far the door is from the bookshelf; and he may raise the question of whether he can fit a piano between them. His question will not necessarily be answered by a catalogue of pianos, however large, which gives the lengths of pianos in feet and inches. For when he sees the distance from the door to the shelf, he does not see it as having a certain length in feet and inches. (Peacocke, 1986, p. 1)

Peacocke's point, of course, is not that the man perceives the distance from the door to the bookshelf in metric units instead, but rather that he perceives the distance without perceiving it in terms of any units whatsoever. Perception is *unit free.* And while being fine-grained is not strong evidence on its own for being analog, being fine-grained *and* unit free surely is. For it is hard to imagine how a non-analog representation could represent indefinitely many magnitude values without the help of units.

Yet Peacocke's assumption that perception is unit free is open to question. While he is undoubtedly correct that perception does not represent magnitudes in imperial, metric, or other familiar units, it is far from obvious that it could not have its own proprietary units. Peacocke's argument to the contrary seems to be based on the fact that such units are not immediately transparent to introspection. But it is not clear why one would expect them to be. As the extensive philosophical debates about the content of perception show (e.g., Hawley & Macpherson, 2011), the path from introspection to content is often subtle and indirect.

One might reply that Peacocke's observation about introspection is supported by the science of perception, which does not appeal to units either. For example, Lee (2017, p. 163) argues that scientific models of duration perception do not appeal to units. But whether perceptual science is committed to units is controversial. For example, Bennett (2011) appeals to perceptual science in arguing that size is represented by eye-level units, which explains why "people often remark that the room that they spent early, childhood years in looks much smaller when revisited later in life" (p. 348). Thus, while Peacocke's observations about introspection are suggestive, the scientific evidence needs to be weighed alongside introspection before a confident conclusion can be reached about whether perception is truly unit free, and there is still disagreement about exactly what the scientific evidence tells us.

2.2 | Dretske and Matthen

According to Dretske (1981), the message that *a* is F is carried by a signal in analog form just in case the signal carries additional information about *a* beyond that entailed by *a*'s being F. Otherwise the message is carried in digital form. For example, the sentence "The cup has coffee" carries the message that the cup has coffee in digital form since it carries no additional information beyond that which the message entails. By contrast, a picture of a cup of coffee will inevitably carry the message that the cup has coffee in analog form since it will also carry information about the cup's color, size, shape, and other properties. Dretske argues that beliefs are like sentences in that their messages are often digital. You can acquire the belief that the cup has coffee without acquiring any beliefs about the cup's color, size, or shape. But perception, Dretske argues, "is informationally profuse and specific in the way a picture is" (1981, p. 142). If you see a cup of coffee, you inevitably see its color, size, and shape as well. Dretske concludes that perception is analog.

One might worry about the extent to which Dretske's observations generalize beyond vision. Even if visual perceptions tend to carry many messages when they carry any, is it so obvious that auditory or olfactory perceptions are packed with information in the same way? If not, Dretske's observation may have more to do with the peculiar richness of vision than with any general truths about perception.

Another worry stems from the idiosyncratic nature of Dretske's definition of analog representation, which falls under neither the continuous conception nor the mirroring conception. Dretske's definition has the counterintuitive consequence that all signals carry analog messages *and* digital messages. For example, the sentence "The letter is an A" not only carries the digital message that the letter is an A, but also the analog message that the letter is either an A or a B. Likewise, a picture of the letter A not only carries the analog message that the letter is an A, but also the digital message that consists of the conjunction of all of its analog messages. For Dretske, a signal or representation is thus never analog or digital full stop, but only relative to this or that message. As Matthen (2005, p. 64) notes in a penetrating discussion of Dretske's view, this generates a tension since Dretske clearly wants to maintain that perception is analog full stop and that belief is digital full stop. Dretske thus seems to presuppose, without explicitly defining, a categorical concept of analog representation. Yet the fact that perception tends to carry many analog messages in Dretske's sense does not show that perception is categorically analog. A long conjunctive sentence also carries many analog messages in Dretske's sense but is not categorically analog in any plausible sense. (See also Peacocke, 1989, p. 315, who points out that Dretske's definition fails to capture the sense in which sentences and judgments are wholly nonanalog.)

While Dretske's definition of "analog" is idiosyncratic, the general picture he has in mind is fairly clear. On Dretske's view, perception is raw and unprocessed, and only gets unpacked ("digitized") into discrete messages in cognition. Matthen (2005) challenges this general picture on empirical grounds. According to Matthen, perception science indicates that sensory information is processed and categorized from the get-go. In the case of vision, such processing begins in the retina. The impression that perception is raw and unprocessed—that is, analog—is an illusion.

> It is true, then, that one cannot attentively see an 'A' without seeing an 'A' of a certain size, colour, and font. This is why the attentive 'A' message seems to be analogue in the sense defined by Dretske: it comes inextricably bound up with information about size, font, and location. However, it would be wrong to conclude that the visual 'A' message has not yet been processed by 'a digital-conversion unit whose purpose is to extract pertinent information from the sensory representation'. What has happened is that the message has been assembled from digitized components in a prescribed form. The attended image seems, from the point of view of the perceiver, to be pictorial in character, to resemble the image projected on the retina… Consequently, the attended image appears, from a subjective point of view, to be concrete and fully specified, and not to have been subjected to a process of categorization. Visual attention creates this illusion by reassembling the messages it has separately devised. (Matthen, 2005, pp. 69–70)

The visual system has separate channels for processing different features—hue, brightness, orientation, etc. When one attends to a specific location or object, the outputs of those separate channels are then reassembled to create a large message. Matthen's suggestion is that perception only seems to be raw and unprocessed because introspection puts us in touch with that finished product.

Given Matthen's observations, should we therefore conclude that perception is nonanalog? Not necessarily. While Matthen is surely right that perception is the product of considerable processing that does not mean it is nonanalog. Analog representations—in either the continuous sense or the mirroring sense—can be the outputs of processing just as surely as digital representations can be.

3 | IMAGERY

3.1 | Mental rotation and mental scanning

A number of psychologists have argued that the mental representations underlying imagery are analog. I begin by describing two classic studies that have been taken to support this conclusion: one concerning *mental rotation* and a second concerning *mental scanning*.

Shepard and Metzler (1971) showed subjects a pair of three-dimensional shapes, and asked them to determine, as quickly as possible, whether they were the same or different by pressing the appropriate key. Taking just those trials on which the shapes were in fact the same and the subjects answered correctly, Shepard and Metzler found that the subjects' reaction times were a linear function of the degree to which the shapes were out of rotation. The more out of rotation they were, the more time it took subjects to answer that they were the same. This result is often interpreted to show that subjects mentally rotate image-like representations of the shapes in their heads (see also Kosslyn, 1980; Metzler & Shepard, 1974; Shepard & Cooper, 1982).

In one of the first mental scanning experiments, Kosslyn (1973) had subjects memorize a drawing of some object (e.g., a boat). The subjects were then asked to close their eyes and call the image up in their "mind's eye," while focusing on one end of it (e.g., the bow). When asked to press a button once they could "see" another part of the object (e.g., the anchor), their response times took longest for parts that were furthest away. This has been taken to show that subjects have image-like representations in their heads that they mentally scan (see also Finke & Pinker, 1982, 1983; Kosslyn, Ball, & Reiser, 1978; Pinker, Choate, & Finke, 1984).

The interpretation of these and other similar results has been highly controversial (for some highlights, see Block, 1981, 1983; Kosslyn, 1980, 1994; Kosslyn, Thompson, & Ganis, 2006; Pylyshyn, 2002, 2003; Thomas, 2018; Tye, 1991). But suppose we take them at face value. In what sense of "analog" might they be taken to support the existence of analog representations? In the remainder of this section I canvass three answers.

3.2 | Kosslyn's functional space

Traditional analog representations were models of what they represented. Thus, one might build an analog representation of an aircraft in flight by placing a model airplane in a wind tunnel. Similarly, mental images might be construed as models of the visible world, and thus as analog representations in this traditional sense. But whereas traditional analog representations retain some of the same basic properties as that which they model (e.g., a model airplane might be the same shape and color as the real airplane it models), it is doubtful that mental images retain the same basic properties as the portions of the visible world that they represent. If you open up my skull and peer into my brain while I imagine a stop sign, you will not see anything that is red and octagonal. Mental images are thus unlike photographs and other familiar images.

But according to Kosslyn, even though mental images are not *literally* spatial, they are nevertheless *functionally* spatial (Kosslyn, 1983, pp. 22–23; Kosslyn, 1994, p. 5; Kosslyn et al., 2006, p. 12). They function *as if* they were laid out in space. For example, if my mental image of a stop sign is stored in the firing patterns of various neural populations in my head, then on Kosslyn's view those firing patterns function as if they constituted an octogonal physical space.

The main problem with this suggestion is that it is far from clear just what a functional space is supposed to be, or how it might be rigorously characterized (Fodor, 2003, pp. 36–37; Palmer, 1978; Pylyshyn, 2002, 2003). For example, it is unclear just what it would take for my neurons to fire "as if" they constituted an octagonal physical space.

At times, Kosslyn attempts to elucidate the concept of a functional space by appealing to an array representation in a digital computer (Kosslyn, 1980, p. 33; Kosslyn, 1994, p. 5; Kosslyn et al., 2006, p. 12). But that blurs the distinction between analog and digital representation. If the representations associated with mental imagery are only functionally spatial in the same sense that representations of digital computers are functionally spatial, it is doubtful that those representations are analog in any interesting sense (Pylyshyn, 1984, p. 204; Pylyshyn, 2003, pp. 359–368).

At other times, Kosslyn appeals to what Fodor (2007) has dubbed the "picture principle": if a representation is pictorial, its parts represent part of what the representation as a whole represents (Kosslyn, 1980, pp. 33–35; Kosslyn, 1994, p. 5; Kulvicki, 2015; Tye, 1991). As Fodor puts it, "Take a picture of a person, cut it into parts however you like; still, each picture-part pictures a person-part" (2007, p. 108). By contrast, consider the sentence, "There is a person with brown hair and blue eyes." Its parts do not all represent part of what the sentence as a whole represents.

But while the application of the picture principle to concrete representations may be relatively straightforward, its application to mental representations is more vexed. The main problem is that the concept of a part is itself most clearly defined for spatial entities, and so the application of the picture principle to mental representations faces a dilemma: either "part" is understood to mean spatial part, in which case we are back to implausibly supposing that imagery representations are literally laid out in space; or else "part" is understood in some looser sense that allows for merely functional parts, in which case the proposal is not a clear improvement over appeals to a functional space.

3.3 | Shepard's second-order isomorphism

Shepard and Chipman (1970) and Shepard (1978) distinguish "first-order" isomorphisms, which concern similarities between a representation and its object, from "second-order" isomorphisms, which concern similarities governing the relations among representations and relations among represented objects. For example, a photograph of a square would instantiate a first-order isomorphism if the portion of the photograph depicting the square were itself a square. By contrast, a mercury thermometer does not instantiate a first-order isomorphism because it does not use temperatures to represent temperatures. It does instantiate a second-order isomorphism, however, since similarities among temperatures correspond to similarities among mercury heights. While Shepard thinks that it would be absurd to suppose that mental images instantiate a first-order isomorphism (an image of a red square is not itself red and square), he takes the results from mental rotation and mental scanning studies to show that mental images do instantiate a second-order isomorphism. Since he takes second-order isomorphisms to be indicative of analog representations, he concludes that mental images are analog.

As Palmer (1978) and Peacocke (2019) observe, the distinction between first- and second-order isomorphisms is unnecessary. An isomorphism is a type of structure-preserving mapping between domains, and as such concerns the relations among items in those domains. What Shepard calls a "second-order isomorphism" is thus really just an isomorphism. But as Palmer also points out, the concept of an isomorphism is too weak to capture what Shepard wants since it can be instantiated by paradigmatically digital representations concerned solely with same–different classifications (see also Pylyshyn, 1984, pp. 204–205). What is needed is a specification of the type of similarity that is characteristic of analog representations. In the following passage, Shepard advances a proposal.

> By an analogical or analog process I mean just this: a process in which the intermediate internal states have a natural one-to-one correspondence to appropriate intermediate states in the external world. Thus, to imagine an object such as a complex molecule rotated into a different orientation is to perform an analog process in that half way through the process, the internal state corresponds to the external object in an orientation half way between the initial and final orientations. And this correspondence has the very real meaning that, at this half-way point, the person carrying out the process will be especially fast in discriminatively responding to the external presentation of the corresponding external structure in exactly that spatial orientation. The intermediate states of a logical computation do not in general have this property. Thus, a digital computer may calculate the coordinates of a rotated structure by performing a matrix multiplication. But the intermediate states of this row-into-column calculation will at no point correspond to—or place the machine in readiness for—an intermediate orientation of the external object. (Shepard, 1978, p. 135; quoted in Maley, 2011, pp. 121–122)

In this suggestive passage, Shepard is apparently trying to articulate a concept of analog representation that fits with the mirroring conception. But there has to be more to the mirroring conception of analog representation than the requirement that intermediate internal states correspond one-to-one to intermediate external states since, as Pylyshyn (1984, p. 203) points out, a series of descriptions of a sequence of events would then count as analog.

3.4 | Maley's covariational account

Drawing inspiration from Shepard, Maley (2011, p. 122) provides a covariational account of analog representation in which "the represented quantity covaries with the representational medium." Or more precisely:

> A representation R of a number Q is analog if and only if:
>
> 1. There is some property P of R (the representational medium) such that the quantity or amount of P determines Q.
> 2. As Q increases (or decreases) by an amount d, P increases (or decreases) as a linear function of $Q + d$ (or $Q - d$) (Maley, 2011, p. 123).

In the case of Shepard and Metzler's mental rotation study, the degree to which the two shapes are out of alignment (*Q*) is represented by some neural property (*P*) such that the latter increases or decreases linearly with the former. Assuming that reaction time, in turn, increases or decreases linearly with the amount of the neural property (e.g., because it takes longer for neurons to fire at a faster rate), this account thus promises to explain the mental rotation results. Similarly, in the case of Kosslyn's mental scanning experiments, we can suppose that the distance between two objects (*Q*) is represented by some neural property (*P*) that increases or decreases linearly with the distance. Again, assuming that reaction time tracks the amount of the neural property, the reaction time data from the mental scanning studies can be explained too.

Maley's account of analog representation is an exemplar of the mirroring conception. Maley is explicit that his account does not require the medium of representation to be continuous. If a boy represents the number of runs his baseball team scores by the number of stones he puts in a bucket, his representation will be analog on Maley's account. Likewise, a representation of the degree to which two shapes are out of alignment will count as analog whether it is coded by the rate at which a population of neurons fires (a continuous quantity) or the number of neurons firing above some threshold within a given population (a discrete quantity).

Although Maley's account of analog representation seems like a step in the right direction, it faces various prima facie difficulties. First, the requirement that *P* and *Q* be related by a *linear* function is surely too strong, and should probably be relaxed to a monotonic function, as Maley suggests in a footnote. Second, the requirement that *Q* be restricted to numbers should also be relaxed. When the brain represents the degree to which two shapes are out of alignment or the distance between two objects, it represents a quantity or magnitude, but not necessarily a number. Third, Maley's definition does not leave room for misrepresentation. The mercury on a malfunctioning thermometer may fail to rise as the temperature increases, and thus misrepresent the temperature. Fourth, Katz (2016) objects that a digital representation could have a property (say, temperature) that has nothing to do with why it represents what it does, but just so happens to covary with the represented quantity (the computer gets hotter as it computes larger sums). Maley's account counterintuitively counts such a representation as analog. Fifth, consider a Turing machine that operates over strokes in unary notation. One stroke represents the number one; two consecutive strokes represent the number two; three consecutive strokes represent the number three; etc. As Schonbein (2014) observes, the strokes on the Turing machine's tape clearly satisfy Maley's analysis, yet the discovery of Turing machines was a key achievement in the theoretical foundations of digital computation. Maley's analysis thus appears to counterintuitively classify the representations of a Turing machine as analog. Sixth, the account only applies to representations of one-dimensional quantities, but it is plausible that representations of multidimensional quantities, such as color space, can be analog too.

Some of these difficulties have easy fixes (surely the first three). Others might be considered features rather than bugs. For example, just as Maley (2011) distinguishes analog from continuous representation, he also takes pains to distinguish digital from discrete representation. He would thus be likely to reject the claim that Turing machines are exemplars of digital computation. They are exemplars of discrete computation, which is not the same thing. Whether and how Maley's covariational account should be amended to address these prima facie difficulties is thus an open question.

4 | NUMEROSITY

4.1 | Numerical discriminations and the ratio signature

Most human beings spend years studying mathematics, at least informally as they learn to count, and then often more formally in the classroom. But even without such an education, it turns out that humans have a primitive ability to represent numerical information (Beck, 2015; Dehaene, 2011; Gallistel, 1990). For example, if you are rapidly presented with two arrays of dots on a screen such that you lack the time to explicitly count them, you can nevertheless reliably determine which array has more dots, even when variables such as area and density are controlled for (Barth, Kanwisher, & Spelke, 2003; Franconeri, Bemis, & Alvarez, 2009). Similarly, if you are rapidly presented with one array of dots on a screen and then a series of aurally played tones, you can reliably determine whether the sum of the dots-plus-tones is more or less than a second array of dots (Barth et al., 2003). These abilities have been demonstrated not only in mathematically educated adults, but also in pre-school children (Barth et al., 2006), 6-month-old infants (Xu & Spelke, 2000), and adults from cultures that lack words for integers greater than three (Gordon, 2004; Pica, Lemer, Izard, & Dehaene, 2004). In fact, a primitive ability to track numerical information has been discovered in a wide range of non-human species, including rats (Church & Meck, 1984), pigeons (Rilling & McDiarmid, 1965), and fish (Agrillo, Daddo, & Bisazza, 2006).

While impressive, these primitive numerical abilities are not without their limitations. Most significantly, the ability to discriminate two numbers is a direct function of their ratio. As the ratio of two numbers approaches 1:1, the ability to reliably discriminate them deteriorates. Thus, if one array has 35 dots and a second array has 40 dots, you are more likely to successfully determine that the second array has more dots than you would be if the two arrays had 40 and 45 dots, respectively. For although the difference in both cases is the same—five dots—the ratio of 35:40 is less than the ratio of 40:45. By contrast, your probability of successfully discriminating 35 from 40 dots will be exactly the same as your probability of successfully discriminating 7 from 8 dots or 70 from 80 dots.

4.2 | Continuous magnitudes

Several authors argue that this ratio signature is evidence that numerical representations are analog in the continuous sense; they have continuous magnitudes as their vehicles rather than discrete symbols (Dehaene, 2011; Gallistel & Gelman, 1992, 2000; Gallistel, Gelman, & Cordes, 2006). As Gallistel et al. (2006, p. 252) point out, when numerosity is represented by voltage levels in an analog computer, "noise in the voltages leads to confusion between nearby numbers." Thus, the ratio signature is readily explained by the assumption that numerical representations have noisy, continuous magnitudes as their vehicles.

By way of analogy, Dehaene imagines that Robinson Crusoe tracks the number of cannibals alighting on his island by channeling water through a bamboo pipe into a hallow log for a fixed amount of time for each person who arrives. Crusoe then uses the height of the water in the log to represent the number of cannibals on the island. Assuming that the water flows through the pipe at a variable rate, Crusoe's system would engender the ratio signature.

> Let us suppose, for instance, that water flow is not perfectly constant and varies randomly between 4 and 6 liters per second, with a mean of 5 liters per second. If Robinson diverts water for two-tenths of a second into the accumulator, one liter on average will be transferred. However, this quantity will vary from 0.8 to 1.2 liters. Thus if five items are counted, the final water level will vary by between 4 and 6 liters. Given that the very same levels could have been reached if four or six items had been counted, Robinson's calculator is unable to reliably discriminate the numbers 4, 5, and 6. If six canibals land, and later only five depart, Robinson is in danger of failing to notice that one of them is missing. (Dehaene, 2011, pp. 18–19)

As Dehaene understands the analogy, the fact that Crusoe uses a continuous magnitude, water height, is crucial to explaining the ratio signature. Thus, Dehaene concludes that nervous systems must use continuous magnitudes too when they represent numerosities. "The nervous system—at least the one that rats and pigeons possess—does not seem to be able to count using discrete tokens" (Dehaene, 2011, p. 19).

Similarly, Gallistel et al. (2006) point out that there would be no reason to expect discrimination to track the ratio of the numbers being compared if numerical representations had discrete symbols as their vehicles.

> For example, the bit-pattern symbol for 15 is 01111 whereas for 16 it is 10000. Although the numbers are adjacent, the discrete binary symbols for them differ in all five bits. Jitter in the bits (uncertainty about whether a given bit was 0 or 1) would make 14 (01110), 13 (01101), 11 (01011), and 7 (00111), all equally and maximally likely to be confused with 15 because the confusion arises in each case from the misreading of one bit. These dispersed numbers should be confused with 15 much more often than is the adjacent 16. Similarly, a scribe copying a handwritten English text is presumably more likely to confuse "seven" and "eleven" than to confuse "seven" and "eight." Thus, the nature of the variability in a remembered target number implies that what is being remembered is a magnitude—a real number. (Gallistel et al., 2006, p. 252)

Two claims should be distinguished. The first, which is common to Dahaene and Gallistel et al., is that the ratio signature is evidence of continuous vehicles. The second, expressed in the final sentence of the passage from Gallistel et al. is that these vehicles have real numbers as their contents.

It is clear from the context that Gallistel et al. take the first claim to support the second. It is because the vehicles are continuous that Gallistel et al. take them to represent real numbers. They seem to reason that continuous vehicles could only represent continuous contents. Laurence and Margolis (2005) object that content is not constrained by format to the extent that Gallistel et al. assume. Just as discrete symbols such as "π" and "$\sqrt{2}$" can represent real numbers, continuous magnitudes can represent discrete values. Thus, even if Gallistel et al. are right that the ratio signature is evidence of continuous magnitudes, we should not conclude that those magnitudes represent real numbers.

There are also grounds to challenge the first claim that the ratio signature is evidence of continuous magnitudes. While continuous vehicles would explain the ratio signature, so would discrete vehicles so long as they stood in the right mirroring relation to the numerosities they represent (Beck, 2015; Maley, 2011). For example, suppose that numerosity were coded by the number of neurons firing above some threshold in a given a population. The number of neurons is a discrete value; a half-neuron cannot fire and a full neuron cannot half-fire. But the number of neurons firing can be subjected to noise just as readily as a continuous magnitude (such as the firing rate of a population of neurons) can be. And if the number of neurons firing were subject to the right kind of noise, comparisons of two numerosities that were based on the number of neurons firing would become less and less reliable as the ratio of those numerosities approached 1:1. The ratio signature would thus emerge. The upshot is that Dehaene and Gallistel et al. conflate the two senses of analog representation that we introduced at the start of this study. While the ratio signature is evidence that numerosity representations are analog in the mirroring sense, it is not evidence that numerosity representations are analog in the continuous sense.

4.3 | The picture principle again

The mirroring conception of analog representation promises to explain the ratio signature, but in what sense do the vehicles "mirror" numerosities? In a passage that is reminiscent of Kosslyn's appeal to the picture principle, Carey (2009, p. 135) writes:

> In analog iconic symbols, such as a realistic picture of a dog representing a dog, parts of the symbol represent parts of the represented entity: the ears on the picture represent the ears of the dog, respecting spatial relations that hold in reality. The word 'dog', in contrast, contains no information about ears or any other part of a dog. Analog magnitude representations are analog in this very sense: the symbol for 3 (———) contains the symbol for 2 (——), respecting the actual numerical relations between 2 and 3.

Carey's proposal here confronts the same dilemma as Kosslyn's. On the one hand, if "part" just means spatial part, then the claim that the vehicle representing a numerosity of two is a part of the vehicle representing a numerosity of three is unjustified. It is certainly *possible* that, say, a numerosity of two is represented by neurons N_1 and N_2 firing and that a numerosity of three is represented by neurons N_1, N_2, and N_3 firing. In that case, the vehicle representing three would "contain" (i.e., have as its spatial part) the vehicle representing two. But another possibility, also compatible with the ratio assumption, is that the firing of more neurons corresponds to a lesser numerosity (Beck, 2015, p. 838). In that case, the vehicle representing two would actually contain the vehicle representing three, and so it would not be true that "parts of the symbol represent parts of the represented entity." Alternatively, numerosities might be represented by neural firing rates, which would also be inconsistent with a spatial interpretation of the picture principle; a firing rate of 20 Hz is not a spatial part of a firing rate of 30 Hz (Clarke, n.d.; Peacocke, 2019). A further concern is that numbers themselves do not have spatial parts (Ball 2017, pp. 131–132; Clarke, n.d.).

On the other hand, if "part" is not interpreted to mean spatial part, it is not clear what it could mean. What is the sense of "part" such that the number two is a *part* of the number three or such that 20 Hz is a *part* of 30 Hz? Perhaps there is an abstract notion of part that could help here—maybe a notion of syntactic part (e.g. Kulvicki, 2015). But it is at least nonobvious how any such notion would apply. Moreover, the notion of part that we reach for should not be so abstract as to apply too widely. For example, if the sentence "Jake bakes" represents Jake and the property of baking, then there is an abstract sense of "part" such that the (syntactic) parts of the sentence represent part of what the sentence as a whole represents (Beck, 2015, p. 838). Yet surely we would not want to count atomic sentences as analog representations.

4.4 | Covariation

We have already seen that a covariational account of analog representation presents as a tempting alternative to the picture principle when it comes to explaining the sense in which mental imagery representations are analog. The same is true of numerosity representations (Beck, 2015; Maley, 2011). If we suppose that some neural magnitude increases or decreases with the numerosity represented, we can explain the ratio effect. Noise in the neural magnitudes will lead them to be confused as their ratio approaches 1:1. Note, moreover, that no assumptions need to be made about whether the neural magnitudes are continuous or discrete since noise will lead to discrimination failures for high-ratio magnitudes in either case. The covariational account thus promises to capture the sense in which numerosity representations are analog according to the mirroring conception.

5 | FUTURE DIRECTIONS

This study has critically reviewed research on analog mental representations in perception, imagery, and numerosity. As we consider where future investigations of analog mental representation might make progress, two features of the foregoing discussion are worth highlighting.

First, whereas philosophers have traditionally focused on perception while considering analog mental representation, psychologists have traditionally focused on imagery and numerosity. As a result, the discussion above was ordered not only by domain (first perception, then imagery and numerosity), but also roughly by discipline, with a review of philosophical work dominating the section on perception and a review of work by psychologists playing a more prominent role in the sections on imagery and numerosity. This alignment is contingent, and points the way towards new avenues of research. In particular, there would seem to be room to bring psychological research more directly to bear on whether perception is analog. In this vein, it is noteworthy that the ratio signature applies not only to discriminations of numerosity, but also to discriminations of a wide variety of perceptual magnitudes, including luminance, loudness, and distance (see any psychophysics textbook on Weber's Law). Thus, it may be possible to construct an analogous argument that perception is analog.

Second, we have seen that there is little agreement about how to define analog representation. The discussions of perception seemed to presuppose the continuous conception or, in the case of Dretske, a *sui generis* conception. The discussions of mental imagery and numerosity, by contrast, seemed to presuppose the mirroring conception, though there was little agreement about how to articulate that conception. This is one place where further philosophical work would be helpful. It would be a mistake, however, to insist that there is one true concept of analog representation that ought to guide future research. Rather, the important thing going forward is to be explicit about which concept is relevant in any given context of inquiry, and to formulate it as rigorously as possible.

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CONFLICT OF INTEREST

The author has declared no conflicts of interest for this article.

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